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Students' Critical Thinking in Analyzing Geometry Problem

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Abstract: The aim of this research is to describe students' critical thinking when analyzing geometry problem at high school students. The indicators of critical thinking used in this research were six indicators adapted from Facione's research. This research subjects were three tenth graders. The result of this research showed that all three subjects have good critical thinking based on six indicators that have been determined by the researchers. It was shown by all their correct conclusions.

Key Words: critical thinking, analyzing geometry problem, self-regulation

Abstrak: Tujuan dari penelitian ini adalah untuk mendeskripsikan berpikir kritis siswa SMA dalam menganalisis soal geometri. Indikator berpikir kritis yang digunakan dalam mendeskripsikan berpikir kritis pada penelitian ini adalah enam indikator yang diadaptasi dari penelitian Facione. Penelitian ini merupakan penelitian deskriptif kualitatif dengan subyek penelitian 3 siswa kelas X. Hasil penelitian menunjukkan bahwa tiga siswa memiliki berpikir kritis yang baik berdasarkan enam indikator yang telah ditentukan oleh peneliti. Hal ini ditunjukkan dengan kebenaran kesimpulan akhir yang diberikan seluruh subjek.

Kata kunci: berpikir kritis, analisis soal geometri, regulasi diri

INTRODUCTION

Ciritcal thinking constitutes as profound and essential ability of individual (Aizikovitsh-Udi & Cheng, 2015). To such a degree, critical thinking must be habituated since early stage. Critical thinking is inseparable from learning process. According to Zhou, et al., (2013), critical thinking locates within cognitive ability and it is essential in learning. Furthermore, Peter (2012) states that critical thinking is a product of learning process. Hence, school must take into account critical thinking ability. School must include critical thinking ability in every learning process in school.

Mathematics lesson in school constitutes as one lesson providing critical thinking ability training for students. Kriel (2013) explains that Mathematics lesson offer critical thinking training through problem-solving approach. As well, Chukwuyenum (2013) has reported that students mathematical ability increases due to critical thinking ability. Then, Aizikovitsh-Udi and Cheng (2015) explain that students' critical thinking ability increases when solving mathematical problem in the daily life. Based on the above-mentioned research results, critical thinking and Mathematics strongly correlates. When performing Mathematics lesson, students automatically train their critical thinking ability. Vice versa, when students train their critical thinking ability, their mathematical ability also increases.

In the context of Mathematics learning in Indonesia, critical thinking ability is included as one of learning objectives. On "Presentation of the Minister of Education and Culture Republic of Indonesia Press Workshop: Implementation of the 2013 Curriculum", mathematics learning is designed to stimulate students' critical thinking in solving problems. In addition, The Decree of Ministry of Education and culture number 69 of 2013 explains that each lesson must contain competencies that take into account students' critical thinking. Furthermore, 21st century skills emphasize on critical thinking as foundation component in each lesson. Thus, it is important to, without exception, include critical thinking in Mathematics learning.

Geometry is one of the sub-topics in mathematics that provides critical thinking skills. According to Sahin (2008), geometry is one of important sub-topic

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in Mathematics. Geometry lesson allows students to think critically, to solve contextual problems, and to employ higher-order thinking skill. Hence, teacher might take into account geometry lesson as one approach to increase students' critical thinking ability.

This paper aims at describing students' critical thinking ability in solving geometrical problems. This paper took six Facione's indicators (2015) in describing students' critical thinking ability, i.e: (1) understanding and writing any obtained information using student's own language (interpreting); (2) identifying the correlation of the obtained information (analysis); (3) assessing statement and information credibilty (evaluation); (4) drawing conclusion from the obtained information (inferencing); (5) presenting convincing and coherence results (communication); and (6) performing self-reflection and assessment (self-regulation). Primarily, Facione's indicators were taken since it is comprehensively developed and it is widely used in numeorus research including research by Hidayanti (2016) and Selviana (2017).

METHOD

This research employed qualitative design and fell under descriptive approach. This research was conducted at SMAN Taruna Nala Malang East Java on the even semester 2017/2018 academic year. It tooks three students of the tenth grade. The three students of the tenth grade were chosen by using purposive sampling. At first, the researchers gave 24 students item to be solved. It was obtained that three students answered correctly and they have good communication. This research employed test and semi-structured interview to obtain the data. The subjects were intervewed right after completing the test. It aimed at preventing external influence when explaining which decreases data validity. The test used is presented in the Figure 1.

The test was used since the subjects were not able to directly determine each side of the triangle. In addition, after obtaining each line of triangle, subject needed to evaluate using cos *C* theorem to check whether the side obtained was correct or not. Data analysis of this study employed Miles and Hubberman (2014) perspectives: data reduction, data presentation, and drawing conclusion.

RESULTS

According to the results, the three subjects satisfy the entire six indicators of critical thinking. The three subjects provided identical conclusion at the end, but during the process in convincing the result, they had different explanation. To obtain deeper regarding the work done by the subjects, semi-structured interview was conducted right after finishing the test. The following is the detailed explanation of subjects' work and interview.

Subject 1 (S1)

The following Figure 2 presents S1 work in doing the test.

ΔABC is a right triangle with right angle in A, the size of angle B is 30°, AD is median toward BC, if Cd is 1 cm and AC 3cm. Determine the perimeter of ΔABC, and explain it!

Figure 1. The Test Used

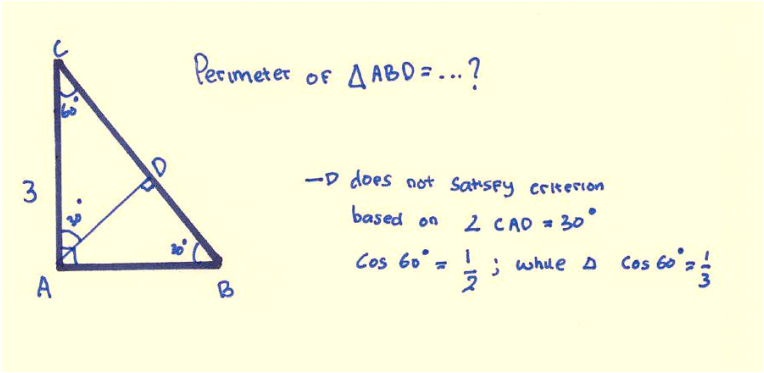

Figure 2. S1's Work

According to the work above, in terms of interpretation, S1 wrote all information known. S1 also drew right triangle based on the problem and provided detailed information on each element. S1 named the right angle as A, and the other angles as B and C. Then, S1 wrote the obtained length of the side, and the size of each angle with mathematical symbols.

In terms of analysis, S1 correlated the obtained information. S1 was able to explain that ABC is right triangle, with A as right angle and B angle is 30° with angle of triangle properties. Then, S1 determined C angle by using formula of $= 180° - 90° - 30° = 60$. Furthermore, S1 correlated cosine theorem into ACD triangle with the obtained length sides of ACD. According to the cosine theorem using ratio of ACD triangle sides, it obtained $\cos C = \cos 60° = \frac{CD}{AC} = \frac{1}{3}$.

The following is the results of semi-structured interview with the subject to obtain deeper the information regarding analysis indicator.

P : You applied cosine theorem on ACD triangle. How do you believe that you can use the cosine theorem in the ACD triangle?

S1 : Because the ACD triangle is a right triangle.

P : How do you know that angle D is right angle. It's not explained in the test questions?

S1 : AD is median, so I assumed it is right triangle

P : Why do you think so?

S1 : In the past I was taught that the median is a perpendicular line. If it is perpendicular, it will form a right angle

In terms of evaluating, S1 connected the previous knowledge about $\cos 60° = \frac{1}{2}$ with the cosine value found in the problem using the ratio of the lengths of the sides of the triangle. It is known that the cos *C* value obtained from the ratio of the sides of the ACD triangle is $\frac{1}{3}$. Based on these two facts, S1 found that there are two different values of cos 60°.

In terms of inferring, S1 concluded that the triangle in the test is undefined. S1 strengthened the argument by saying that there is a difference in the cosine value at an angle of 60° between the results of calculations using a ratio of the sides of the ACD triangle with the value of. Thus, S1 concluded that the triangle was undefined and the problem could not be solved.

In terms of communicating and self-regulation, S1 was well-performed. In terms of communicating, S1 wrote the entire information, explanation, and reasons to strengthen the conclusion. As well as the self-regulation, S1 performed no mistake when writing information until drawing conclusion.

Subject 2 (S2)

The following Figure 3 presents S2 work in doing the test.

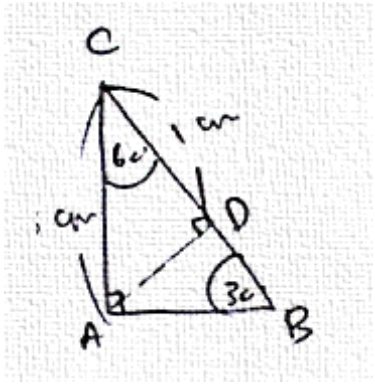

Figure 3. S2 Work in Writing Question Information

Based on the figure 3 above, in terms of interpretation, S2 wrote down the entire information in the form of drawing and explanation. S2 drew right triangle ABC with the right angle is A, according to the question. Then, S2 drew right angle AD line between BC sides and wrote the angle size was 30°. In addition, S2 also wrote down the length of triangle sides in the drawing.

P : Why did you write all the information in the form of pictures?

S2 : Because by making drawings, it is easier for me to understand what was asked about Sir. Besides that, by drawing a picture, I can more easily determine what I need to do first to find the perimeter of an ABC triangle

In terms of analysis, according to the interview, S2 stated that by drawing a pircture of triangle, S2 was easier to determine the information explained in the question and further it eased S2 to determine the perimeter of an ABC triangle (See figure 4).

In terms of evaluating, S2 correlated the obtained information to determine new information. To determine the length of BC and AB, S2 took the size angle of B. According to the calculation made in Figure 4, the length of BC is 6 cm. Then, S2 determined the

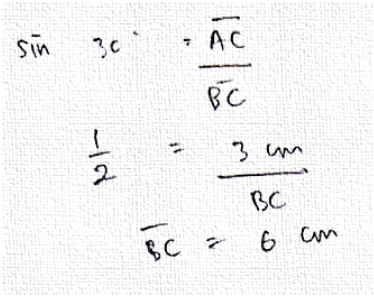

Figure 4. S2 Work in Processing Information

length of AB by employing cosine theorem toward the angle of B (See Figure 4).

Based on calculation made by S2 shown in Figure 5, it obtained that AB is $3\sqrt{3}cm$. In addition, S2 also determined the size angle of C by using angle of triangle concept. Based on the calculation, S2 obtained the size angle of C is Then, after obtaining the new information from the calculation, S2 determined the perimeter of triangle by summing the entire size length. It obtained that the perimeter of the riangle is $(9 + 3\sqrt{3})cm$ the calculation made by the S2 is presented in the following Figure 6.

In terms of evaluating, at first, S2 failed to satisfy the criterion since S2 directly determined the perimeter of the triangle without re-checking the validity of the information obtained. S2 initially calculated the the perimeter of ABC triangle and determined $(9 + 3\sqrt{3})cm$. However, after undergoing self-regulation process, S2 re-checked the answer by using cosine theorem on the angle of C. Based on the results obtained in the previous indicator, S2 has found that the angle C is 60°. By using the cosine concept, S2 found that there was a difference in the cosine value between the value obtained from the previous concept and the value obtained using a comparison of the sides of a triangle. According to the previous knowledge, the value of $\cos 60° = \frac{1}{2}$ while cos 60° obtained from the ratio of triangle sides was $\frac{1}{3}$. Based on these findings S2 found a discrepancy between the facts and the knowledge previously obtained.

In terms of inferring, S2 initially deduced that the perimeter of the triangle ABC can be determined. But after going through a process of self-regulation, S2 changed the conclusion, the triangle cannot be formed. S2 concluded that there were differences in the cosine values of C angles based on prior knowledge and the results obtained using a comparison of the sides of a triangle (see Figure 7).

In terms of communication, S2 has written down all the information and the conclusion correctly. In

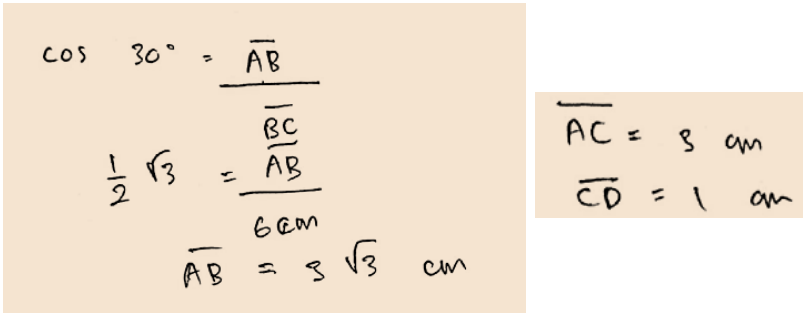

Figure 5. S2 Work in Processing Information

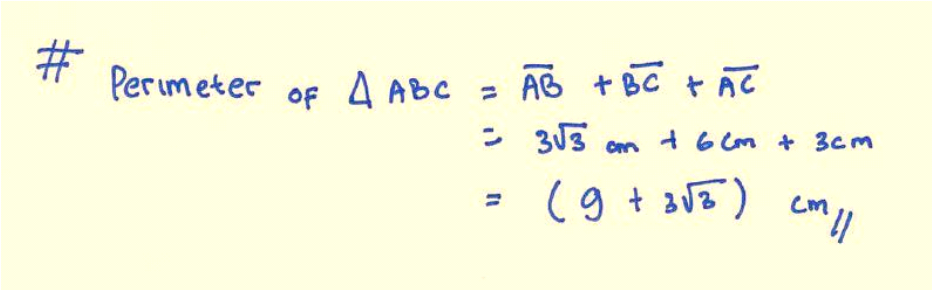

Figure 6. S2 Work in Processing Information

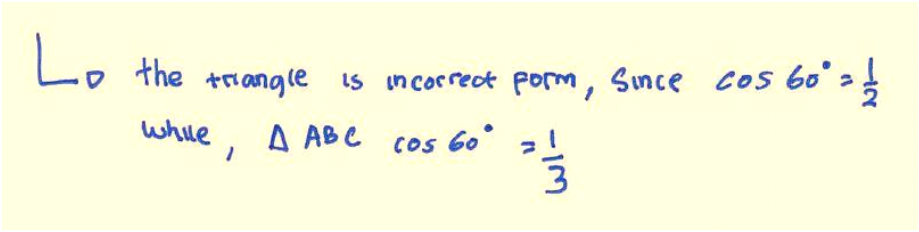

Figure 7. S2 Work in Evaluating

terms of self-regulation, S2 has done well. This can be seen from the revised answer given in Figure 7. S2 immediately wrote that the triangle is wrong or cannot be formed because the value of 60° was different based on previous knowledge with the value found through comparison of the sides of different triangles. The following are the results of the interview of the researcher (P) with S2 related to the process of self-regulation.

P : What was in your thought when reading the question?

S2 : I thought the problem was easy at first, sir, I was only asked to find the perimeter. So right away, I began to calculate. Then I thought about why there was something strange, you don't usually give questions this easily, there must be something hidden. he..he..he .. And it turns out right sir, it turns out that the triangle was wrong.

P : How can you say that the triangle is wrong?

S2 : The cosine value just isn't right, which means the triangle is wrong. So I cant determine the perimeter.

P : What do you think about the problem you analyzed earlier?

S2 : The problem is tricky sir, I rarely get questions like this.

In terms of self-regulation, based on the interview, S2 initially assumed that the question given was easy. Then, because the teacher often give questions that demand critical thinking, the S2 began to think again and re-examine the answers that have been written. Based on the results of re-examination carried out, S2 was able to improve the conclusions.

Subject 3 (S3)

The following Figure 8 presents S3 work in doing the test.

In terms of interpretation, based on Figure 8, S3 wrote all the information in the problem in the form of

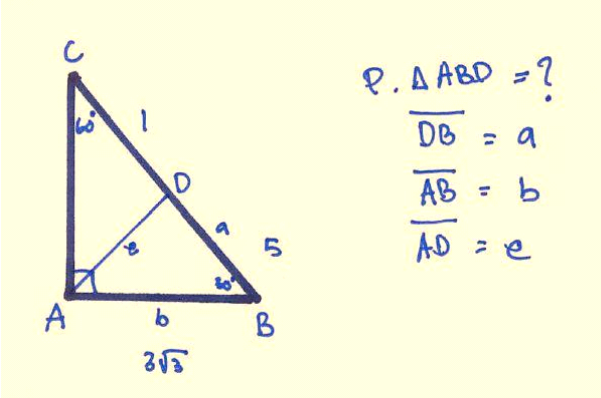

Figure 8. S3 Work in Processing Information

pictures and details. S3 drew a right triangle ABC with a right angle at A, according to the problem. In addition, S3 also wrote some information to determine the perimeter of a triangle. S3 wrote three information, namely $BD = a$, $AB = b$, and $AD = c$.

In terms of analyzing, S3 correlated the information obtained to find new information. S3 determined the angle C using the concept of the number of angles in a triangle. Based on calculations performed by S2, the angle C angle is 60°. S3 then used the information about the angle C compare between ABC, DAC, and DBA triangles.

Based on Figure 9, using the Pythagorean Theorem on the ACD triangle, it obtained information that $AD = 2\sqrt{2}cm$. Then, from the comparison of DAC with DBA it obtained $BD = 8cm$. After obtaining the length of BD and AD, S3 usds the Pythagorean Theorem to find the length of AB. Based on calculations, S3 obtained AB length is $6\sqrt{2}cm$. Based on new information that has been obtained by S3, S3 determined

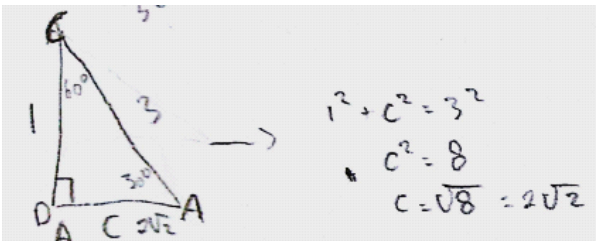

Figure 9. S3 Work in Processing Information

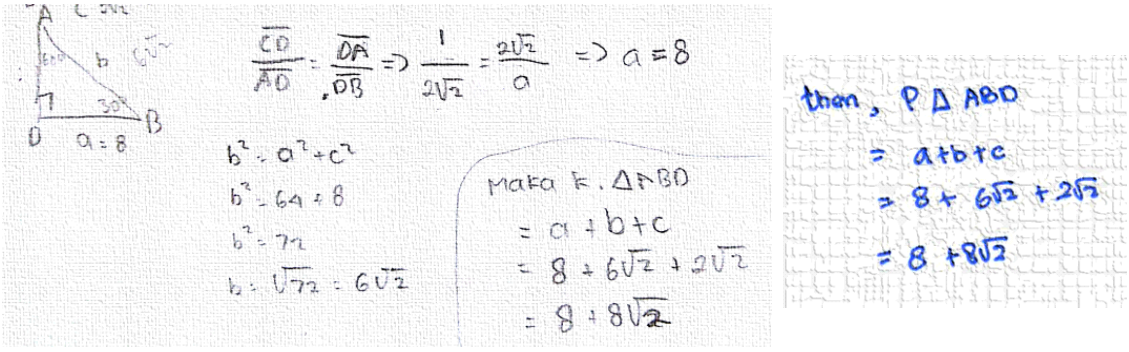

Figure 10. S3 Work in Processing Information

the perimeter of the triangle ABC by adding up the three sides and it is found that the perimeter of the triangle is $(8 + 8\sqrt{2})cm$. The results of these calculations can be seen in Figure 10.

In terms of evaluation, (Figure 10), S3 initially failed to meet the indicator because S3 directly determined the perimeter of the triangle without checking the accuracy of the information obtained. S3 initially calculated the perimeter of the triangle ABC at first and found that the perimeter is $(8 + 8\sqrt{2})cm$. However, after going through a process of self-regulation, S3 re-checked the answers obtained. S3 re-checked the answer by using the cosine concept at angle C. Based on the results obtained in the previous indicator, S3 has found that the angle C is 60°. By using the cosine concept, S3 found that there is a difference in the cosine value between the value obtained from the previous concept and the value obtained using a comparison of the sides of a triangle. Based on previous knowledge, the value of $\cos 60^\circ = \frac{1}{2}$, while the value of cos 60° obtained from the comparison of the sides of a triangle is $\frac{1}{3}$. Based on these findings, S3 found a discrepancy between facts and knowledge previously obtained.

In terms of inferring, S3 initially deduced that the perimeter of the triangle ABC can be determined. But after going through a process of self-regulation, S3 changed the conclusion that the triangle does not meet the trigonometric rules but the perimeter can be determined. S3 concluded that there were differences in the cosine values of C angles based on prior knowledge and the results obtained using a comparison of the sides of a triangle. The conclusion of S3 can be seen in Figure 11.

In terms of communication, S3 has written down all the information and the conclusion correctly. This

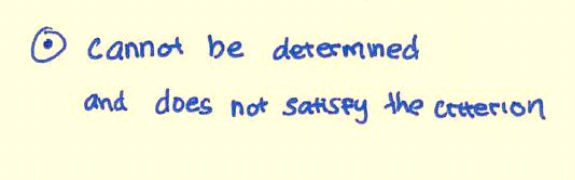

Figure 11. S3 Conclusion after Undergoing Self-Regulation

can be seen from the revised answer given in Figure 11. S3 wrote that the triangle is wrong or cannot be formed because the value of cos 60° was different based on previous knowledge with the value found through comparison of the sides of different triangles. The following are the results of the interview of the researcher (P) with S3 related to the process of self-regulation.

P : What was in your thought when reading the question?

S3 : I think this is a test for middle school kids, sir. Judging from the triangle, the length of each side can be determined using congruence. Later, when I obtained the value, I just determined the perimeter.

P : Then why do you wrote "can be determined, but does not meet trigonometric rules"?

S3 : Indeed, the value can be determined, sir, only the triangle is wrong because it does not meet trigonometric rules.

P : Which trigonometric rules do you mean?

S3 : Cosine rules sir. The cosine value of angle C does not correspond to the cosine value of angle

P : Are you sure that the value that you find using a comparison will be the same as the value that you find using trigonometry?

S3 : Sure, Sir.

P : Did you try it?

S3 : Not yet, Sir

P : Do you often work on problems like this.

S3 : No, Sir

In terms of self-regulation, based on the results of interviews with S3, S3 concluded that the triangle ABC does not satisfy trigonometric rules. However, S3 also wrote that the perimeter of the ABC triangle can be determined. S3 also responded confidently that the perimeter found must be correct. However, because S3 has not yet determined the perimeter of ABC in another way, S3 was not sure of the answer itself.

DISCUSSION

In evaluating, S1 chose to use information related to the trigonometric function: cosine to evaluate the initial information provided. The selection of this information was based on an S1 understanding of the cosine concept. S1 compared the cosine value of C angle found by comparing the length of the sides of the triangle with the cosine value that has been studied previously. S1 found that there are differences in cosine values even though the angles should be the same. This finding supports the results of research conducted by Umraatin (2012) that the understanding and mastery of a lesson by students influences the thought process carried out.

In the second indicator, S2 and S3 obtained new information related to the length of each side needed to determine the perimeter of the ABC triangle. Both subjects were able to obtain this information from the results of reading the questions and then linking them with prior knowledge. This reinforces the results of previous studies that by reading questions, ideas can be generated (Talun, 2015; Arifani et al., 2017) and linking new information with previous information can generate new ideas (Jesica, 2016). In addition, the research results of Hidayati et al. (2017) explain that ideas can be generated by reading questions and linking with prior knowledge also corroborates the results of research that has been found. However, S2 and S3 made mistakes.They were initially unable to satisfy the second indicator. On the results of working on the test questions, it can be seen that S2 and S3 immediately determined the length of the sides of the triangle without checking the credibility of the information presented in the questions.

Based on the results of the interview, it can be concluded that the two subjects are only fixated on the information provided and then immediately received the information without checking the truth. S2 and S3 also explained that they rarely worked on questions and exercises that train their critical thinking. If it is not trained routinely, students will have difficulty in critical thinking. This is in line with the opinion of Snyder and Snyder (2008) who say that the lack of critical thinking skills is caused by too much memorization, having less thinking, having limited mastery of concepts, students are not given the practice of critical thinking, and the time is too short. In addition, according to Peter (2012) lack of exercise results in low student abilities.

CONCLUSION

All three subjects have good critical thinking based on indicators that have been determined by the researchers. In the first indicator (interpretation), the three subjects were able to rewrite the information provided using pictures; making it easier for them to do data processing. In the second indicator (analysis), all subjects have been able to find the key information that is the size of the C angle and the cosine value of the C angle. In the third indicator (evaluation), all subjects have been able to determined the value of the C cosine using the cosine value that has been obtained from previous knowledge. In the fourth indicator (inferring), all subjects concluded that the triangle given in the problem actually could not be formed. All subjects agreed that there is a difference between the cosine value of C angle which is found through the comparison of the sides of the triangle with the cosine value that has been obtained from previous knowledge. In the fifth indicator (communication), all subjects wrote down all the information provided in the form of pictures and detailed conclusion about the triangle coherently and clearly. In the sixth indicator (self-regulation), all subjects performed good self-regulation. This is proven from the work done by the subjects. At the beginning of the work, there were two subjects who made mistakes on the second and third indicators. However, through good self-regulation, both subjects were able to realize the mistakes that have been made and corrected the errors to produce the proper conclusions.

Based on the results of the study, it is suggested to conduct research on the application of a learning method that is able to improve students' analysis and evaluation skills. One learning method that researchers suggest is problem-based learning methods. This is because problem solving improve critical thinking.

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