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Student Mathematical Representation Ability with Reflective Cognitive Style in Solving Geometric Problems

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Abstract: This research aims to describe the mathematical representation ability of students with reflective cognitive style in solving geometric problems. The subjects of this study were two students whose reflective cognitive style were selected based on the results of the MFFT test. The research method used was descriptive research with a qualitative approach. The ability of mathematical representation was described based on three standards of mathematical representation ability, namely; (1) creating and using representations to organize, record, and communicate mathematical ideas, (2) choosing, using and translating between representations to solve problems, (3) using representations to create models and interpret mathematical, physical, and social phenomena. The results showed that subjects with reflective cognitive style can use mathematical representation capabilities well from various types of representations, namely visual images, verbal written texts, and mathematical expressions.

Key Words: mathematical representation ability, reflective cognitive style, geometric problem

Abstrak: Riset ini bertujuan untuk mendeskripsikan kemampuan representasi matematis siswa bergaya kognitif reflektif dalam menyelesaikan masalah bangun datar. Subjek penelitian ini adalah dua siswa yang bergaya kognitif reflektif yang dipilih berdasar hasil tes MFFT. Metode penelitian yang digunakan adalah penelitian deskriptif dengan pendekatan kualitatif. Kemampuan representasi matematis dideskripsikan berdasar tiga standar kemampuan representasi matematis, yaitu; (1) membuat dan menggunakan representasi untuk mengorganisasikan, mencatat, dan mengkomunikasikan ide-ide matematika, (2) memilih, menggunakan dan menerjemahkan antar representasi untuk menyelesaikan masalah, (3) menggunakan representasi untuk membuat model dan menginterpretasi fenomena matematis, fisik, dan sosial. Hasil penelitian menunjukkan bahwa subjek dengan gaya kognitif reflektif dapat menggunakan kemampuan representasi matematis dengan baik dari berbagai jenis representasi, yaitu visual gambar, verbal teks tertulis, dan ekspresi matematis.

Kata kunci: kemampuan representasi matematis, gaya kognitif reflektif, masalah geometris

INTRODUCTION

In Government Regulation No. 32 of 2013 concerning National Education Standards, it is stated that the process of learning mathematics requires the development of logic and mathematical abilities of students, the ability to choose and use the steps of each process in solving problems (Departemen Pendidikan Nasional, 2013). According to the National Council of Teachers of Mathematics (2000) mathematics learning involves five standard processes, namely problem solving, reasoning and proofing, communication, connections, and representation. In the five standard processes, the ability of representation is used to organize, record, and communicate ideas, as well as facilitate students in solving problems to develop students' mathematical understanding flexibly and precisely.

Representation according to Luitel (2002) is a process of constructing mathematical knowledge. The construction process is carried out by students with other individuals or groups. The results of Jitendra et al.'s (2016) research conclude that representations can be used effectively to teach mathematical problem solving. In this regard, Hudiono (in Sabirin, 2014) representations are concepts that give certain meanings and play as an activity, for example the activity of solving students' mathematical problems can first

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interpret the problems given into meanings that are easily understood by students. Hence, the ability of representation is an interpretation of a problem or concept in the form of an easy interpretation in describing the solution of a problem or concept.

The ability of mathematical representation is essential for solving mathematical problems. According to Montague (dalam Fadillah, 2009), successful and precise problem solving is possible using appropriate representation. In making mathematical problem solving, it requires solving strategies, metacognitive activities, experiences, and mathematical abilities not only prioritized knowledge. One of the mathematical abilities used in the process of solving is mathematical representation. Therefore, the ability of representation is needed in the process of solving mathematical problems.

Problem solving activities are used to determine students' thinking processes and abilities in solving mathematical problems. Problem solving activities involve thinking, understanding, processing, and analyzing the information contained in the problem. The process is influenced by the thinking style of each student, one of the differences is the accuracy and speed in processing the information in the problem; called cognitive style. According to Desmita (2009), cognitive style is a character possessed by individuals in thinking, remembering, processing information that is consistent and lasts long, solving problems, making decisions, and organizing. Cognitive style of each individual is different, there are individuals who take short time in processing a problem but the accuracy and truth is not necessarily accurate. Conversely, there are individuals who need a long time to process a problem but it is accurate.

Woolfolk and Margetts (2016) distinguish cognitive styles more specifically including: (a) field dependent– independent field, (b) impulsive-reflective, and (c) verbal imagery-nonverbal imagery. Of the several types of cognitive styles that are put forward, impulsive-reflective cognitive styles are the focus of this study since they are related to students' accuracy in solving problems. The influence of reflective-impulsive cognitive style with the level of errors made by students. Research by Soemantri (2018) confirms that the faster students solve problems the higher the level of errors committed, while the longer students solve problems the less likely the mistakes made. This means that students with a reflective force make fewer mistakes than students with an impulsive force.

Rozencwajg and Corroyer (2005) also suggest that the two styles are the nature of cognitive style related to time in decision making and performance in solving problems that contain high levels of uncertainty. Thus this research was interested in examining the ability of student representation in solving mathematical problems based on cognitive style. This problem is very important to be studied, since learning mathematics will be more emphasized on problem solving activities to answer the demands of mathematical skills in the technology era 4.0.

METHOD

This research used a qualitative approach and descriptive qualitative type. The subject of this research was Eight Graders at Al-Ma'arif 02 Malang Islamic Middle School. The research instruments consisted of: (1) MFFT test sheets adopted from Warli (2010), (2) wide flat area problem student sheet. In addition to the mathematical problem sheets, in this study also used interviews containing questions about problem solving resulting from the subject's thought. The Figure 1 presents a mathematical problem used.

Pak Arif owns a quadrangle plot of land. He intends to divide it into six part. Each will be a rectangle with 7 m. Determine the perimeter of each!

Figure 1. Mathematical Problem

RESULTS

The study was conducted in Eight Graders at Al-Ma'arif 02 Malang Islamic Middle School, in April 2019, which was started by giving MFFT to 24 students. Based on MFFT results, students were grouped in impulsive or reflective cognitive learning styles. Students exhibiting reflective cognitive styles were then given the task of solving problems. The following are the results of grouping MFFT student tests based on student work.

In Figure 2, it is found that from 24 students there were 12 slow inaccurate students, eight reflective students, four impulsive students, and no studen was categorized as fast accurate students. Then, the group chosen for analysis was the reflective cognitive style.

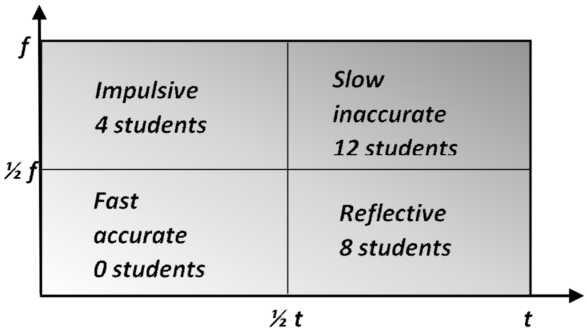

Figure 2. MFFT Results on Student Cognitive Style

Based on the results of the MFFT, two students of reflective cognitive style were chosen to solve the mathematics problem. The following are the results of the problem solving tests of the two reflective students, they were HN and RF.

HN Subject

According to Figure 3, the first step performed by HN was writing the information on the problem "perimeter = 7 m" and drew a square divided into six equal parts. In stage 1, HN presented the problem information using verbal representations in the form of written text and visual representations. At this stage, HN showed a good representation ability and the representation used was appropriate to the problem.

In stage 2, HN started to identify a solution by trial and error. HN employed the information around the rectangle. The perimeter of the known rectangle was 7 m, HN made a representation of the image for the rectangle being inserted and estimates the length and width of the rectangle thus the circumference was 7 m. This situation indicated that HN understood the concept of perimeter. After finding the right rectangle size, HN then correlated the results with the main problem, which was square area. From the results, HN claimed the intended size. In this step, HN calculated the area of the intended square.

Experiments carried out by HN is presented in Figure 2 section 2. Firstly, HN determined the length 2.5 and width 1, and obtained the circumference of 7 m. Then, he checked the area of the square. The calculation of the area of a square started with calculating the length of the other sides, which was 1 × 6 = 6 (multiplied by 6 since the square was divided into 6 equal parts). From the lengths of the sides, a square area of 2.5 × 6 = 15 was obtained. But this was not the area of the square in question because the length of the sides of the square should be the same. In the second experiment, HN determined the lengths and

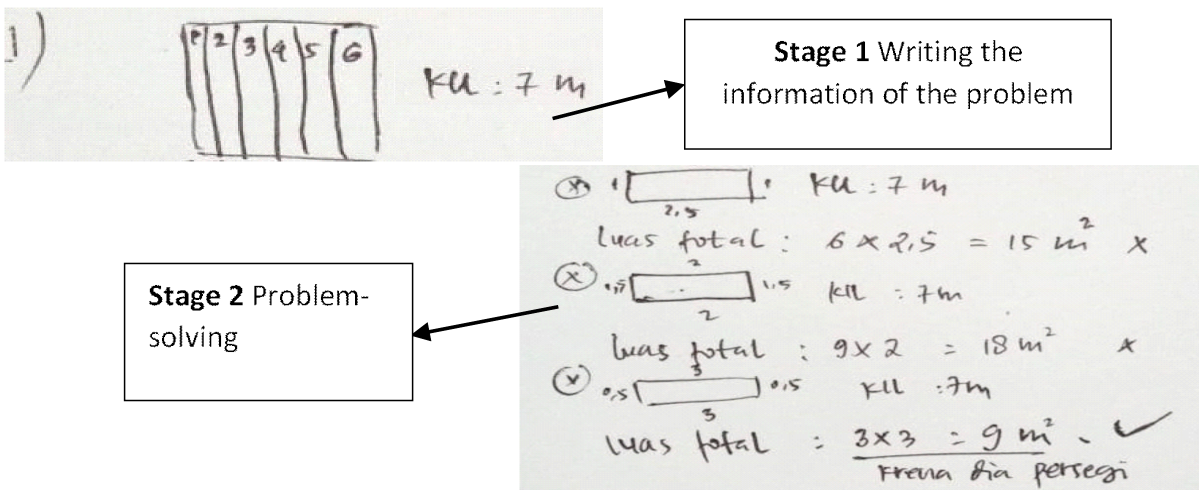

Figure 3. The Problem-solving Results of HN

widths of 2 and 1.5, and obtained a circumference of 7 m. Using the same process, HN checked the area of the square based on the lengths of the selected sides. HN calculated the area of a square by first calculating the length of the second side of the intended square, which was 1.5 × 6 = 9 (multiplied by 6 since the square was divided into 6 equal parts). From the length of these sides, then HN calculated the area of the square, which was 2 × 9 = 18. But HN realized that this was not the area of the square because the size of the sides of the square should be the same. HN tried the third attempt and determined the length of 3 and width of 0.5 and obtaibned the perimeter of 7m. Then, HN checked the area of the square if the length or first side is 3, then the other side of the square is 0.5 × 6 = 3 (multiplied by 6 since the square was divided into 6 equal parts), then HN obtained 3 × 3 = 9, this is the correct area of a square because the size of the sides of the square is the same. Therefore, in the third experiment HN believed that the area of the square or land is 9 m^2. For more details, researchers conducted interviews with the following results:

P : *How did you solve the problem?*

HN : *I tried to determine the length and width which meets the perimeter size. Then, I consider that the square must have exact identical size for each shape. Then, to make it easy for the calculation and to adjust with the information in the problem, I drew the shape every time I try to determine the size. Finally, I made it in the third time I try.*

P : *What information did you obtain about the perimeter and the area size?*

HN : *Perimeter of a square is the total length (distance) of the boundary of a square; it has 2 length and 2 width. Area is a flat occupied within the boundary of a square. The problem asked the area of the square, then the calculation is* $side_1$*$side_2$. *It means that the size is exacty similar. Thus, in this problem, I need to pay attention whether the side is exactly similar or not, if it is no, then it is not a square.*

In stage 2 (see Figure 3), HN used a mathematical and visual representation of an image to find the right solution. The mathematical representation used by HN greatly helped HN to create a solution in accordance with the concepts he already had and the problem information. Based on the results of the interview, it was also seen that HN was able to convey verbally what he had done in the problem solving process. Based on the two methods used by HN, if viewed from the three standard of mathematical representation processes according to NCTM can be concluded as follows:

1. Expressing mathematical ideas by making and using representations
   HN could make and use the representation seen in step 1 of figure 3 and which serves to record the information contained in the problem and also communicate ideas that will be used for problem-solving.
2. Resolving the problem with selecting and translating between representations
   HN in stage 2 of the initial part of Figure 3 could select and use representations to start solving problems either by trying on problems.
3. Creating mathematical models and interpreting physical and social phenomena by using a representation
   HN in step 2 of figure 3 used the visual representation of the image to make it easier to conduct experiments and find solutions.

The explanation above shows that HN had a good profile of mathematical representation ability to find the right and correct problem solving.

RF Subject

Problem-solving by RF is presented in Figure 4. In stage 1, RF wrote information on the problem by writing “square: 6 parts, perimeter = 7 m”. It is presented by the RF in verbal representations in the form of written text. In stage 1, RF wrote that the land is square, and is divided into six parts visible on “square: 6 parts”. However, RF did not state explicitly that the land is divided into six equal parts, meanwhile a square can be divided into six unequal parts in terms of their size. Furthermore RF wrote “perimeter = 7 m” as one of the instructions to solve the problem.

Stage 2 done by RF was using a visual representation of a square. It was divided into six equal parts, thus the key information was not written in step 1. However, it did not mean that he did not know the information. Presumably, RF understood the information but he did not present it explicitly. The use of appropriate and correct visual representation enabled RF to understand the problem better and find a way to solve. After drawing a square which was divided into six equal parts, RF described one part of a rectangular shape. Mathematical symbols written by RF

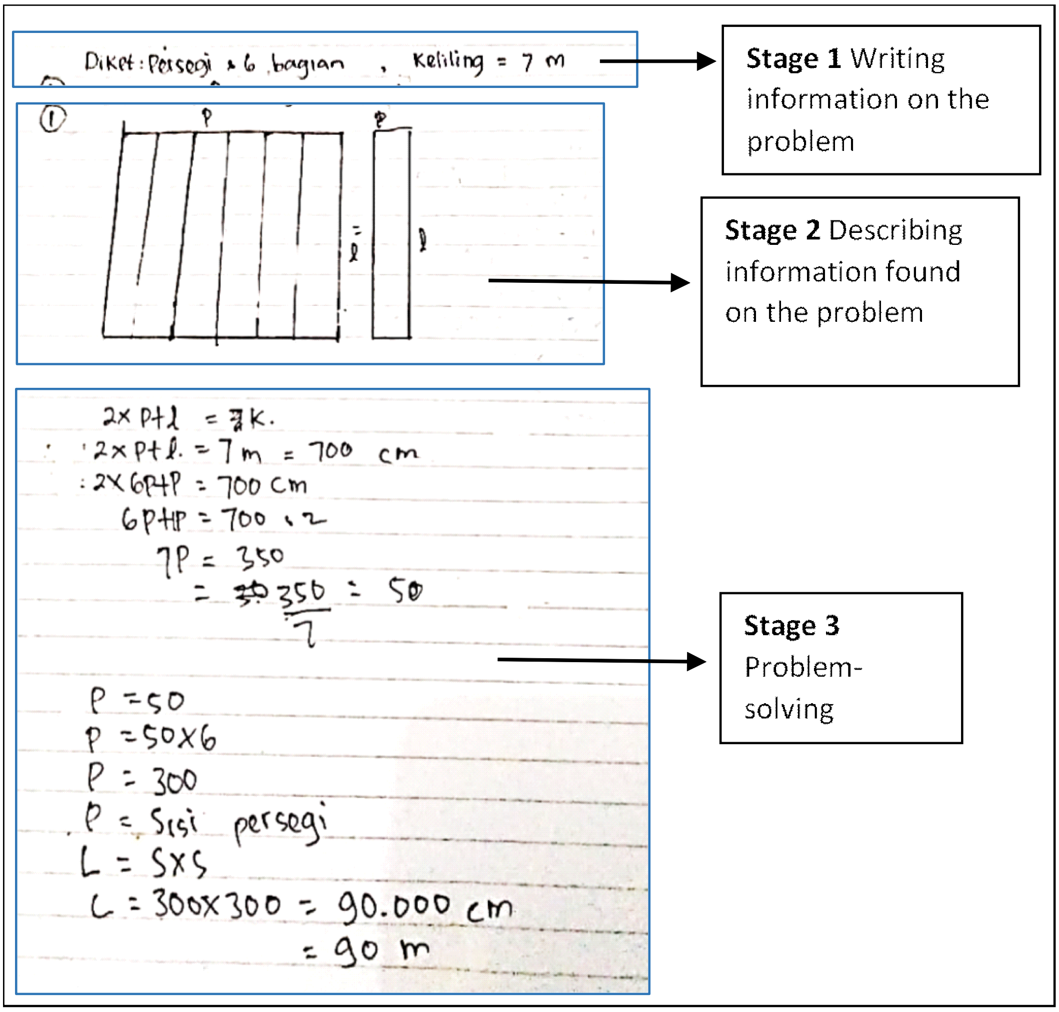

Figure 4. The Problem-solving Results of RF

showed that he understood the path of solving problems related to length and width. The researcher further clarified through interviews as follows:

P : *What did you understand from the question number 1?*

RF : *A square divided into six equal parts, with the perimeter of 7 m*

P : *Try to read again and check what did you miss?*

RF : *No, Ma'am*

P : *Is a square divided into 6 equal parts or not?*

RF : *It is divided into six equal parts and I did not write it*

P : *Is the drawing make you easier to understand the problem?*

RF : *Yes, Ma'am it is. I can express what I thought in the drawing while understanding the problem to find out the solution.*

P : *If it helps you, how did you draw ?*

RF : *I drew a square, then it is divided into six equal parts. From the problem I obtained that the perimeter is 700 m, then I drew one rectangle which then I solve it with the perimeter of rectangle.*

P : *Okay, then what you write "perimeter=700 m" is a perimeter of a square or a rectangle?*

RF : *A rectangle as a divided part of a square. Yes, I understand that I should write one part of the divided square of the perimeter of the rectangle.*

Then in stage 2, RF started to think about solving the problem by using a rectangle perimeter with a length and width according to the interview above. At this stage 3, RF used mathematical expression representations to find solutions. First, RF remembered that the perimeter of a rectangle can be expressed in $2 \times (l + w)$. Second, RF changed the size of the width of one rectangle with p, so that the width of the six rectangles was *6p*, because RF understood from the picture that the width of the rectangle was six times the length since the shape of the rectangle was divided into six equal parts.

Therefore, RF continued to calculate the length. Third, after finding a length that was 50 cm, and since was the length of the side of the square, he replaced the with 50 cm and he obtained 300 m of side size. Lastly, RF calculated the area of a square of 90,000 cm or 90 m. Based on the two solutions made by HN when viewed from the three standard mathematical representation processes according to NCTM, it can be concluded as follows:

1. Expressing mathematical ideas by making and using representations RF could create and used representations (seen in step 1 in Figure 4) that serve to record information that there are both problems and also communicate ideas that will be used for resolution.
2. Resolving the problem with selecting and translating between representations RF (seen in stage 2 of Figure 4) used visual representation of images to translate information and find a solution.
3. Creating mathematical models and interpreting physical and social phenomena by using a representation RF (seen in stage 3, Figure 4) used mathematical symbolic representation to solve problems until finding the correct solution.

The explanation above shows that the RF subject had a good mathematical representation ability to find the correct problem solving, but there are several stages of the error in understanding the problem.

Reflective cognitive style students in this study solved well the given mathematical problems and used the ability of representation in the resolution process. Students with reflective styles used a variety of ways to solve problems because reflective students requires a long time to think and work with various problem solving. This is in accordance with the results of the study which states that reflective student answers can vary and differ so as to indicate that reflective students understand the problem well (Shoimah, Lukito, & Siswono, 2018). Cognitive style plays a role in processing information and analyzing a problem (Nur & Nurvitasari, 2017), especially reflective cognitive style. Reflective students are much more careful, tend to pay attention to accuracy, and take more time to reflect on a situation (Nietfeld & Bosma, 2003).

Reflective cognitive style students in this study, in the process of understanding the problem and expressing information on the problem employed a diverse representations with verbal written text and visual images. This is shown by students writing after understanding the problem in the form of drawing according to the information. They could describe a rectabgle which was then divided into six equal parts so that each rectangular section corresponds to the problem information. This is in accordance with Wiryanto (2014) who states that when students understand a concept, they are able to represent the concept properly in any form whether it is picture or symbolic objects.

In the process of presenting information about mathematical problems, the subjects used verbal representations of written and visual texts that were seen in the results of student work. In terms of verbal representations, reflective students wrote the information provided in the problem. Meanwhile, in the visual representation, they drew the land in accordance with information. Therefore, it seems that the students are able to recognize important information. According to Wijaya, Marja, Michiel, and Alexander (2014) students who are unable to recognize important information in the problem lead to misunderstanding. In this process, the students were very careful in writing or describing information since to avoid further issues in solving the problem.

Furthermore, the students' problem solving process used mathematical expression representation. They wrote down the steps even though there were some errors on the results due to miscalculation. The settlement process was carried out with a variety of strategies in the form of trial and error and also using a rectangle equation. The various ways of problem-solving done by the students indicate novelty of completion.

The results showed that the students considered all possibilities before deciding on completion. Reflective students considered all alternatives before making decisions in situations that have a difficult solution. Both had different paths of completion with each line of thought with the same and correct answers. This shows that reflective students have flexible characters and fluency. According to Siswono (2006) students' ability to use a variety of different ways of solving problems refers to being flexible, while diversity of students in answering problems correctly refers to fluency.

Furthermore, in the interview process, the reflective students were able to explain each process of completion, both when using verbal representations in the form of written text, visual images, and mathematical expressions. The results of interviews can reinforce the results of answers written by reflective students, because reflective students tend not to re-examine the results of the completion before being collected so an interview was needed. When giving answers during an interview, reflective students tended to take a long time to answer, but in the end the verbal explanation was more accurate and complete. In accordance with Philip (in Rahmatina, Sumarmo, & Johar, 2014), before explaining from interview questions, reflective students tended to think of various possible answers according to the context of the question.

CONCLUSION

Based on data analysis and discussion, students with a reflective cognitive style have a good ability to solve problems. This ability was demonstrated by their ability to show the standard representation process: (1) being able to express mathematical ideas by making and using appropriate representations, (2) being able to solve problems by selecting and translating between representations, and (3) being able to create models and interpret mathematical, physical, and social phenomena using representations.

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