

STUDENT MATHEMATICAL COMMUNICATION IN SOLVING TWO VARIABLE SYSTEM OF LINEAR EQUATIONS

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ARTICLE INFO

Article history:

Received 12/10/2020

Approved 11/12/2020

Keywords:

Communication mathematical

Problem mathematics

Solution to problem

Linear Equation

Variable

ABSTRACT

Abstract: Communication is a basic skill for students in mathematics. This study describes students' mathematical communication to solve mathematical problems. The results show that high-cognitive-abilities student are able to consolidate and organize mathematical thinking through the communication of students with moderate and low-abilities. All subjects use mathematical language to present mathematical ideas. High -cognitive student is able to communicate mathematical thinking correctly to friends, teachers and others. All subjects have not been able to analyze and evaluate other mathematical ideas and strategies.

INTRODUCTION

Communication is one of the essential skills that students must possess. NCTM asserts that communication is essential to mathematics (NCTM, 2000). Mathematical aptitude is evaluated in part based on pupils' communication skills. In modern global culture, the capacity to interact well with others is essential for life (Di Giacomo et al., 2013). Students must have great communication skills in order to properly address mathematical problems (Bicer et al., 2013).

Communication and mathematics education are intricately intertwined. Mathematics instruction is intrinsically communicative (Kleden et al., 2015). When the learning material incorporates some form of mathematical communication, the process of transmitting information from knowledge and experience occurs between fellow students and instructors and students. Students are encouraged to convey their ideas, hypotheses, thoughts, and mathematical solutions through communication, resulting in a more developed mathematical process (Viseu & Oliveira, 2012).

Mathematical communication is the interchange of mathematically related information (Masrukan et al., 2015). Mathematical communication is a strategy or approach for transforming ideas that aids students' comprehension (NCTM, 2000). Through mathematical communication, students' ideas can be commented on, discussed, improved, and developed. In addition, the communication process can assist pupils acquire a deeper understanding of concepts and generalize or explain them. Providing students with opportunity to convey their mathematical concepts is a crucial element in fostering their conceptual understanding of mathematics (Vasileiadou, 2013).

Mathematical communication is a person's ability to explain their thoughts and is responsible for listening, interpreting, questioning, and communicating one idea to another when solving problems in class and discussion groups (Clark et al., 2005). When students learn and perform mathematics-related activities, they are able to explain their mathematical concepts, relate them to real-world situations, make connections, and construct new knowledge (Prayitno, 2013). The term "communication" refers to the process of transmitting information from one person to another through signs or words (Son, 2015). In addition, mathematical communication can promote the correct use of mathematical terminology, enabling students to accurately communicate their understanding of mathematical concepts (Suharta, 2019).

Mathematical communication may be vocal (spoken) or written (writing) (Fachruraz, 2011). Communication begins with the process and concludes with the capacity to communicate information orally and in writing with reasonable precision (Caballero et al., 2011). Through writing, kids can generate ideas, comprehend material, and summarize its significance (Santos & Seman, 2015). This study studies the written communication of students, which is supplemented by interview-based spoken communication.

Caselin's research indicates that only a small proportion of students exhibit effective mathematical communication skills (Caselin et al., 2013). The majority of students have difficulty relating the obtained difficulties to real-world circumstances, and their inability to use the knowledge contained in the questions impedes the next phase of the completion process. Other studies have found that, on average, high school pupils had difficulty solving written problems (Dyana, 2018; Edy & Tandilling, 2012; Pramestasari, 2017;

Ramdhani, 2012). According to the initial observation of the researchers, students of MTs Darussalam have inadequate communication abilities. Students' problems expressing their opinions, their propensity to deliver succinct responses, and their reluctance to communicate their thoughts all contribute to the lack of mathematics-related communication among students.

The relationship between communication and problem-solving is inextricable. When students tackle problems involving specific resources, they gain the ability to generate solution ideas and then communicate the rationale for picking the optimal solution technique. Students who are able to effectively explain mathematics concepts demonstrate a good knowledge of concepts and the ability to solve problems based on their mastery of the concepts (NCTM, 2000). By encouraging them to describe how they arrive at their solutions and discuss their thought processes, mathematics communication can aid students in strengthening their mathematical reasoning (Kostos, 2010). Strong mathematics communication abilities enable students to develop a variety of representations, which facilitates the identification of multiple solutions to mathematical problems.

Students engage in a number of cognitive activities as they solve problems, including linking previously learned knowledge and experiences to develop new knowledge (Yeo, 2009). In this scenario, students must develop a consistent mode of thought and approach the problem or assignment as a puzzle to be solved. Cognitive and emotional parts of students' problem-solving processes interact (Furinghetti & Morselli, 2009).

Through problem-solving, teachers can identify difficult issues for students and are familiar with the characteristics of obstacles that match to students' thought processes. This knowledge enables the instructor to characterize the challenges encountered by pupils and their underlying causes. Schoenfeld (Schoenfeld, 2002) argues that problem solving is the basic objective and method of mathematical instruction.

According to some mathematicians, issues are only opportunities to hone one's skills in connection to the most current topic (Haghverdi & Wiest, 2016; Yew et al., 2017). When attempting to solve a problem, students must use and combine mathematical principles in a way that makes the problem challenging (Tural, 2012; Xenofontos, C., & Andrews, 2014). Moreover, Polya (1973) defines four phases of problem-solving, including the following: understanding the problem, which includes understanding the problem's components, such as the obtained information, the questions posed, and the conditions; Implementing solutions, that is, confirming the correctness of previously implemented steps in solving issues and proving their correctness; and re-examination, that is, testing the correctness of previously achieved problem-solving stages.

Junior high school students study the Two Variable Linear Equation System among other things. Numerous studies indicate that students continue to struggle with the Two Variable Linear Equation System. According to Widiyanti's research, pupils get a limited understanding of Two Variable Linear Equation System (Widiyanti, 2011). According to additional studies, a significant proportion of students continued to struggle with appropriately answering questions (Juliana & Jafar, 2017). Based on these issues, the authors conducted a study on the mathematical communication of junior high school students in addressing problems involving two variable linear equation systems.

METHOD

This study employed a qualitative research method with a descriptive approach, as the researcher was attempting to explain or describe students' mathematical communication when completing math problems. Researchers served as data collectors and instruments, therefore their presence on the scene is crucial. This study's research approach comprises of identifying the subject, creating instruments, collecting data, analyzing data, and drawing conclusions. The subjects chosen were three eighth-grade students of MTs Darussalam Malang. The categorization of students' math skills as high, medium, or low was based on daily test scores and teacher feedback. Students with a score between 86 and 100 possessed exceptional math skills. The category of moderate mathematical abilities consisted of students with a score between 70 and 85, whereas the category of low mathematics abilities consisted of students with daily test scores below 70. Three students, designated S1, S2, and S3, were chosen as research subjects based on the outcomes of student evaluations and input from the instructor. S1 is a student with exceptional ability who participates actively in class learning activities. S2 is a student with moderate abilities, whereas S3 is a student with poor abilities who is less engaged in classroom learning activities. Researchers collected this information from teachers who instruct pupils.

At the stage of instrument production, the Problem Solving Test for Two Variable Linear Equation System was the research instrument. The data collection phase consisted of administering tests on the Two Variable Linear Equation System questions that had been prepared for the study topic, followed by an evaluation of the student's work. In addition to analyzing student work, the researcher conducted in-depth interviews with students to assess their grasp of mathematical topics. In the data analysis phase, the results of student work and interview findings were assessed with reference to the modified communication indicators. This stage identifies students' mathematical communication in problem solving. The stage of drawing conclusions involves drawing conclusions based on the data that has been analyzed by the researchers.

The following is the problem test given to students.

"Adi, Berta, and Cici will purchase two sorts of books: notebooks and sketchbooks. Adi purchased seven notebooks and five sketchbooks for IDR 37,500. Betta spent Rp 34,000.00 on four notebooks and six sketchboo. Cici purchases ten books. Cici's funds total is Rp 27,000.00. According to Adi, Cici's funds are insufficient to purchase ten books. Berta claims that Cici's funds are sufficient to purchase ten books, depending on the type of book being purchased. Whom do you believe expressed the accurate viewpoint? Explain your rationale!

RESULTS

In this study, mathematical communication was described using indicators adapted from the National Council of Teachers of Mathematics, specifically in terms of: integrating and structuring mathematical ideas through communication; communicating mathematical ideas or concepts in a manner that is easily understood by others. Colleagues, professors, and other individuals; explain and evaluate the various mathematical strategies and concepts adopted by others; and effectively express mathematical concepts using mathematics language. The researcher then developed sub-indicators for each proposed indicator of the NTCM. In addition, the researchers categorized these sub-indicators into four stages of problem-solving (Polya, 1973). Written mathematical communication, specifically the results of student work, as well as interviews between researchers and students will be detailed. This section describes the mathematical communication of students.

Mathematical Communication of Problem-Solving Data for Student 1 (S1)

When S1 understood the situation, the subject could record the facts gathered and the questions posed using the correct terminology. Figure 1 illustrates an example of S1 work.

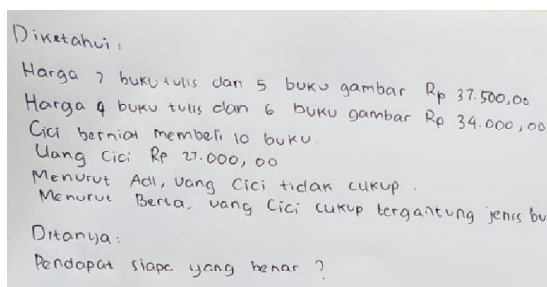


Figure 1. Information Acquired by S1

Based on the outcomes of S1's work, it is evident that S1 was able to accurately record the collected information. S1 is also already aware of the question's intent. The researcher also validated to S1 the responses students provided to the questions.

Q : "What information did you get in the question?"

S1 : "The price of seven notebooks and five picture books is thirty-seven five hundred rupiah. The price of four notebooks and six sketchbooks is thirty-four thousand rupiah."

Q : "Is that all? Or any other information?"

S1 : "Cici intend buy 10 books. Owned money cici twenty seven thousand rupiah. According to Adi, Cici's money no will enough for buy 10 books. Whereas according to Berta's money cici enough for buy 10 books depends type purchased books".

Q : "What is being asked about?"

S1 : "Who gave correct opinion? _ How many price book write and book pictures?"

S1 has written down mathematical concepts using terms and symbols, although they are not yet correct. S1 let x = notebook and y = sketchbook. The right answer would be x = the cost of one notebook and y = the cost of one sketchbook. The illustration below is an example of an S1 work.

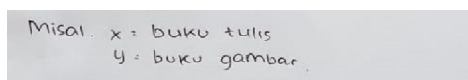


Figure 2. The example of S1

The following is an excerpt from the researcher's interview with S1 when it was confirmed about the error in the sampling.

Q : "What do you write about the x and y variables?"

S1 : "notebooks and sketchbooks"

Q : "Are you sure about your answer?"

S1 : "Hmm ... I think yes Ma'am"

Q : "What are you looking for in the question?"

S1 : "x and y Ma'am"

Q : "Even though you said earlier that x represents notebooks and y represents sketchbooks, it means that you are looking for notebooks and sketchbooks?"

- S1 : "Not Ma'am, but the price"
 P : "It means that your example is not correct, x should show the price of one book and y will show the price of one book"
 S1 : "Oh yes ma'am"

Based on the work of S1 and interviews, at the stage of understanding the problem, S1 could fulfill mathematics indicators: organizing and consolidating students' mathematics thinking through communication. When planning problem solving, S1 can write down mathematics models correctly and completely. The results of S1's work are shown in the following figure.

$$\begin{aligned} 7x + 5y &= 37500 \\ 4x + 6y &= 34000 \end{aligned}$$

Figure 3. Mathematics Model by S1

When the researcher confirmed S1 about the mathematical models that was made, S1 was able to explain it correctly. The following is an excerpt from the researcher's interview with S1.

- Q : "How did you get the mathematical model?"
 S1 : "From what is known in the question Ma'am "
 P : "Please explain how the mathematical model was obtained!"
 S1 : " The price of seven notebooks and five skecthbooks is thirty-seven hundred rupiahs, so $7x + 5y = 37500$. The price of four notebooks and six skecthbooks is thirty-four thousand rupiahs, so $7x + 6y = 34000$."

S1 compile and mathematical model based on the existing problems. The mathematical model created was $7x+5y=37500$ and $7x+6y=34000$. S1 wrote a plan for solving the problem solving it by elimination and substitutions. Based on S1's work and interviews, it appears that S1 could use mathematics terms when making plans for completion. S1 could state the reasons for the mathematical models made. S1 could also write a settlement plan, by using the method of substitution and elimination to obtain the solutions. Mathematical communication indicators that appear when S1 performs the stages of planning problem solving are organizing and consolidating students' mathematics ideas with communication, and using mathematics language in conveying mathematics ideas correctly.

When carrying out the plan, S1 was able to write the steps for working on the problem correctly and completely, by using a combined method of elimination and substitutions. The calculations made are correct. Figure 4 shows the results of S1 work.

$$\begin{array}{r} 7x + 5y = 37500 \quad | \times 4 \\ 4x + 6y = 34000 \quad | \times 7 \\ \hline 28x + 20y = 150000 \\ 28x + 42y = 238000 \\ \hline -22y = -88000 \\ y = \frac{-88000}{-22} \\ y = 4000 \end{array}$$

$$\begin{aligned} y = 4000 &\rightarrow 7x + 5y = 37500 \\ 7x + 5(4000) &= 37500 \\ 7x + 20000 &= 37500 \\ 7x &= 37500 - 20000 \\ 7x &= 17500 \\ x &= \frac{17500}{7} \\ x &= 2500 \end{aligned}$$

$$\begin{aligned} 2500 \times 10 &= 25000 \\ 17000 - 25000 &= 2000 \end{aligned}$$

Figure 4. Steps to complete S1 at the stage of implementing the plan

S1 can write down the steps in solving the problem and the reasons. S1 can also explain the process in completion. This was confirmed by the researcher through the following interview excerpts.

- Q : " After making a mathematical model, how do you do it? Pleas explain the step you did for completing question this ? "
 S1 : "I am eliminated the equation 1 with 2 Maam."
 Q : " What? Why did you eliminate equation 1 and equation 2?"
 S1 : " To obtain the value of y"

S1 compile and mathematical model based on the problems received, in order to obtain two equations: equation (1) and equation (2). S1 then was finding for the value of y first by eliminating the equation (1) and equation (2). After the x value was obtained, then S1 substituted the y value into equation (1) thus the x value was obtained. S1 can write down the steps in solving problems coherently

and correctly when working on questions. At the time of carrying out the completion plan, S1 obtained the result that the value of x was 2500 and the value of y was 4000. The results obtained were correct.

S1 can use mathematics symbols when writing the stages of problem solving and the reasons. Next, after getting the values of x and y , S1 wrote that $25000 \times 10 = 250000$. The following Figure 5 shows S1's work.

Handwritten work of S1:

$$2500 \times 10 = 25000 \quad 27000 - 25000 = 2000$$

Cici bisa membeli 10 buku

Figure 5. S1 Work

When the researcher asked the purpose of S1's writing, S1 stated that it was a calculation of books that Cici could buy with Rp. 27,000.00. S1 wrote that Cici could buy ten books. S1 did not mention the second alternative answer: Cici could buy nine notebooks and one sketchbook with the money she had. The math communication indicator shown by S1 when implementing the plan is to communicate students' mathematics ideas correctly and understandably to teachers, friends and other people. S1 can also apply mathematics language in conveying mathematics ideas appropriately.

At the re-examination stage, students have not changed the mathematics symbols obtained into problem situations. Figure 6 below shows the results of S1's work.

Handwritten conclusion of S1:

Cici bisa membeli 10 buku
Pendapat yang benar adalah pendapat Berta

Figure 6. Conclusion S1

The student has gotten the values of x and y , but have not changed these values to a problem situation: the price of one notebook and the price of one sketchbook. The student has written the conclusions obtained that the correct opinion is Berta's opinion, but students did not write down the reason for solving the problem.

Mathematical Communication of Problem-Solving Data for Student 2 (S2)

When understanding the problem, S2 did not write down the information obtained and the questions asked using words. Figure 7 shows the work of S2.

Handwritten information written by S2:

x : buku tulis
 y : buku gambar
* Cara mengetahui harga buku tulis dan buku gambar dengan eliminasi dan substitusi

Figure 7. Information written by S2

S2 started the work by writing down mathematics ideas using mathematics terms and symbols. S2 wrote x was notebook and y was sketchbook. The information made by S2 was not correct. This error was similar to S1. The correct information should be x = the price of one notebook and y = the price of one sketchbook.

When planning problem solving, S1 can write down mathematics models correctly and completely. The results of S2's work are shown in the following figures.

Handwritten mathematical model made by S2:

Model Matematika
Aci : $7x + 5y = 37500$
Berta : $4x + 6y = 34000$

Figure 8. Mathematical model made by S2

When the researcher confirmed S2 about the mathematical models that was made, S2 was able to explain it correctly. The following is an excerpt from the researcher's interview with S2.

Q : "How did you get the mathematical model?"

S2 : "I'll provide an example, Ma'am, then make it according to the question"

P : "Please explain how the mathematical model was obtained!"

S2 : "The price of seven notebooks and five sketchbooks is thirty-seven five hundred rupiahs, so $7x + 5y = 37500$. The price of four notebooks and six sketchbooks is thirty-four thousand rupiahs, so $7x + 6y = 34000$."

S2 compiled a mathematical model based on the existing problems. The mathematical model created was $7x+5y=37500$ and $7x+6y=34000$. Based on interviews and the results, it can be seen that S2 could use mathematics terms when making plans for completion. S2 can state the reasons for the mathematical models made.

When carrying out the plan, S2 can write down the steps in solving problems that are given correctly but are less complete than S1. The calculation done by S2 is correct. The following figure 9 illustrates S2 work.

$$\begin{array}{l}
 \text{Adi} = 7x + 5y = 37500 \\
 \text{Berta} = 4x + 6y = 34000
 \end{array}
 \begin{array}{l}
 \times 6 \\
 \times 5
 \end{array}
 \begin{array}{l}
 42x + 30y = 225000 \\
 20x + 30y = 170000 \\
 \hline
 22x = 55000 \\
 x = \frac{55000}{22} \\
 x = 2500
 \end{array}$$

$$\begin{array}{l}
 7(2500) + 5y = 37500 \\
 5y = 77500 - 17500 \\
 5y = 20000 \\
 y = \frac{20000}{5} \\
 y = 4000
 \end{array}$$

Figure 9. Steps to complete S2 at the stage of implementing the plan

S2 constructs a mathematical model based on the existing problems in order to obtain two equations denoted by the numbers (1) and (2). (2). $7x+5y=37500$ in equation 1, and $7x+6y=34000$ in equation 2. The first equation is multiplied by six, and the second equation is multiplied by five. After determining that the y coefficients in equations 1 and 2 are identical, S2 determines the value of x by eliminate equations (1) and (2). (2). After obtaining the x value, S2 substitutes the y value into equation (1) to obtain the y value. S2 can correctly write the steps necessary to solve the problem while working on it. S2 obtained the result that the x value was 2500 and the y value was 4000 while carrying out the completion plans. S2 obtained accurate results.

S2 has changed the values of x and y to the problem situation, namely the price of one notebook and the price of one skecthbook, at the stage of looking back. This means that S2 can map mathematical symbols to real-world problems. Figure 10 illustrates a part of S2's answer .

Jadi harga buku tulis 2000
 harga buku gambas 4000

Figure 10. S2 Work

S2 stopped at the sentence stating the price of notebooks and skecthbooks. S2 has found the values of x and y but cannot use the results obtained to answer the question. S2 has not answered the question about the correct opinion of Adi or Berta. S2 did not write conclusions and reasons for solving problems.

Mathematical Communication of Problem-Solving Data for Student 3 (S)

When understanding the problem, S3 did not write down the information obtained and the questions asked using words. S3 only wrote that Adi=37500 and Berta=34000. When confirmed by the researcher, S3 said that this was something that was known in the problem. Similar to S2, S3 immediately started the work by writing down mathematical ideas using terms and symbols. S3 wrote that $x =$ notebook and $y =$ skecthbook. The example made by S3 was not correct. This error was similar to S1 and S2. Whereas correct statement should be $x =$ price of one notebook and $y =$ price of one skecthbook. The following figure illustrates the S3's work.

Adi = 37500
 Berta = 34000
 Misal buku tulis = x
 buku gambas = y

Figure 11. Work done by S3

When planning a problem solving, S3 could write a mathematical model correctly and completely. The results of the S3 work are shown in the following figure.

$$\begin{array}{l} 7x + 5y = 37500 \\ 7x + 6y = 34000 \end{array}$$

Figure 12. The mathematical model created by S3

When asking about how to obtain a mathematical model, S3 was confused. S3 did not give a reason how he obtained the mathematical model. Based on the results of S3's work, it can be seen that S3 can use mathematical terms when making a completion plan, but S3 has not been able to state the reasons for the mathematical model made.

When implementing the plan, S3 multiplied the first equation twice, and the second equation three times. When the researchers confirmed the reason for doing this, S3 seemed confused. S3 did not yet understand the concept of elimination in completing the problem. Figure 13 shows the work of S3.

$$\begin{array}{l} 7x + 5y = 37500 \\ 7x + 6y = 34000 \end{array} \quad \begin{array}{l} \times 2 \\ \times 3 \end{array} \quad \begin{array}{l} 14x + 10y = 75000 \\ 12x + 18y = 102000 \end{array}$$

Figure 13. The Work of S3 when Implementing Problem

Because S3 did not obtain a solution from the system of linear equations, then S3 did not obtain the correct values of x and y. S3 cannot write the solution and the reason for the problem solving step.

Furthermore, S3 did not change the mathematical symbols obtained into the problem situation. S3 also did not write down the conclusions obtained along with the reasons. This is because in the previous stage, the stage of implementing the plan, S3 was still unable to find the values of x and y.

DISCUSSION

Mathematical communication is a means of sharing ideas, thoughts, and concepts in order to enhance student comprehension (NCTM, 2000). The research indicates that kids with high, medium, and poor talents have distinct communication skills. The following describes students' mathematical communication when solving problems with Polya. solution steps (Polya, 1973).

When S1 comprehends the situation, he or she is able to verbally record the information gained and the questions posed. S1 developed informational inquiries that cover situational queries and knowledge gleaned from situations (Erling et al., 2016). The questions and information asked are designed to ensure that the respondent understands the problem posed in the question. In mathematics, the writing process can aid students in synthesizing meaning, comprehending mathematics, and developing mathematical concepts (Santos & Semana, 2015). In contrast to S1, neither S2 nor S3 recorded the information obtained nor the questions asked in written form. Students in grades S2 and S3 include mathematical symbols directly into their written responses to questions. S1, S2, and S3 perform identical operations on the problem data using the symbols x and y. The example of S1 and S2 are a representation used to aid student comprehension of the problem. One of the components of communication is representation (Prayitno, 2013). Even though they have completed the equations, S1, S2, and S3's equations are incorrect; x should be the price of one notebook and y should be the price of one sketchbook.

S1, S2, and S3 have been able to develop a mathematical model based on the current condition at the time of developing a plan of completion (planning). Students are able to convey mathematical ideas by symbol, number, or the link between the two; hence, the idea can be communicated. Becomes beneficial for both herself and others. S1 and S2 were able to describe why the mathematical model was created, however S3 was still unable to explain why the mathematical model was created. S1 and S2 were able to draft the completion plan accurately at the time of its creation. S1 and S2 select a settlement plan utilizing substitution and elimination. In addition, S1 and S2 students tend not to provide justifications for using the completion plan. S1 and S2 were able to provide the proper response during interviews, but they did not write it down because they were not accustomed to doing so. S1 and S2 may employ mathematical terminology while developing a settlement strategy. S3 was unable to compose a settlement strategy and the causes for utilizing the completion plan.

In the process of carrying out the completion plan, S1 and S2 start the completion step by eliminating one of the variables contained in the problem. S1 eliminates the y variable first to get the value of x. Meanwhile, what S2 does is eliminate the x variable at the beginning to get the y value. The results obtained by S1 and S2 are the same and the result of solving the problem is true. S1 tends to be more complete when writing problem solving compared to S2. While S3 still cannot write down the solution and the reason for solving the problem. This is because S3 still does not understand the steps in completing SPLDV. When solving a problem in mathematics, students need good communication skills in order to solve the problem well (Bicer et al., 2013).

In the process of checking back, S1 is not change symbol mathematically obtained _ into _ situation matter . This can be seen from the work of S 1 which writes the final answer in the form of symbols x and y. S1 can write the conclusion of solving the problem and

it is true, but S1 does not write down the reason explicitly. While S2 already changed the values of x and y to the problem situation, namely the price of one notebook and the price of one sketchbook. S2 stops at the sentence stating the price of notebooks and picture books. S2 has found the values of x and y but cannot use the results obtained to answer the question. S2 has not answered the question about the correct opinion of Adi or Berta. S2 does not write conclusions and reasons for solving problems. The writing should present a description and reason for this to happen, present the basis of the conclusions that have been made, and fulfill the requirements asked by the question. While the S3 mathematical communication stage of re-examining (*looking back*), S3 is not change symbol mathematically obtained _ into _ situation matter . S3 also did not write down the conclusions and reasons students wrote the answers. This is because at the previous stage, namely at the stage of implementing the plan, S3 was still unable to find the values of x and y .

Based on the results of research conducted, indicators of mathematical communication according to NTCM that appear when students work on SPLDV problems are as follows. In the process of understanding the problem, S1 is able to organize and strengthen mathematical ideas by using communication. While these indicators have not appeared in S2 and S3. In the planning process, S1 and S2 are able to organize and consolidate mathematical thinking through communication, while in S3 these indicators have not appeared. S1, S2 and S3 are able to apply mathematical language in conveying mathematical ideas appropriately at this stage. In the process of implementing the plan, S1 and S2 have been able to communicate students' mathematical thinking correctly to others. S1 and S2 are also able to apply mathematical language in conveying mathematical ideas appropriately. However, both S1, S2 and S3 have not been able to analyze and assess the mathematical strategies and mathematical ideas of others. In the re-examination process, only S1 is able to communicate mathematical ideas correctly to teachers, friends and other people.

S1 and S2 initiate the completion stage of the completion plan by deleting one of the variables contained within the problem. S1 initially eliminates the y variable to obtain the x value. In the meantime, S2 eliminates the initial x variable to obtain the y value. Results obtained by S1 and S2 are identical, thus the solution to the problem is correct. S1 students are typically more thorough in their problem-solving writing than S2 students. While S3 is still unable to record the solution and rationale for addressing the challenge on paper. This is because S3 can not comprehend the procedures required to complete SPLDV. (Bicer et al., 2013). In order to address a mathematical problem effectively, students must possess strong communication abilities.

In the process of checking back, S1 is not changed from a symbol obtained mathematically to a situational matter. This is evident from S1's work, which represents the final result with the symbols x and y . S1 is able to write the correct conclusion after answering the problem, but does not properly state why. While S2 has already modified x and y to reflect the problem circumstance, namely the price of one notebook and the price of one sketchbook, x and y remain unchanged. S2 halts at the price of notebooks and children's books. S2 has determined the values of x and y but is unable to use them to answer the query. S2 has not responded to the query regarding the right assessment of Adi or Berta. S2 does not write problem-solving conclusions and justifications. The paper should include a description and explanation of why this occurred, the rationale for the conclusions drawn, and fit the requirements of the question. During the S3 mathematical communication stage of re-examining (*looking back*), S3 does not convert the symbol mathematically gained into a different scenario. Also, S3 did not record the students' conclusions and justifications for their replies. This is due to the fact that S3 was unable to find the values of x and y at the preceding stage, namely the stage of plan implementation.

Indicators of mathematical communication according to the NTCM that occur when students solve SPLDV questions, as determined by research findings, are as follows: Through the use of communication, S1 is able to arrange and strengthen mathematical concepts while comprehending the challenge. While these markers are absent from S2 and S3, S1 and S2 are able to arrange and consolidate mathematical thought through conversation during the planning phase, however S3 lacks these indicators. At this stage, S1, S2, and S3 are able to appropriately utilize mathematical language when communicating mathematical concepts. In the course of implementing the strategy, S1 and S2 have been able to accurately convey the mathematical reasoning of their students to others. S1 and S2 are also capable of effectively communicating mathematical concepts using mathematical language. However, neither S1, S2, nor S3 are able to study and evaluate the mathematical tactics and ideas of others. During the reexamination process, only S1 is able to correctly express mathematical concepts to teachers, peers, and other individuals.

CONCLUSION

S1 can record facts and questions when understanding a problem. S1 can write questions using mathematical symbols. Planning-stage S1 can write a completion plan. S1 did not write down the strategy's rationale, but she explained them verbally. S1 is better at recording plan implementation completion and reasons than S2 and S3. S1 may compose solutions using mathematical symbols. S1 did not change the mathematical symbols after reexamination. S1 can write a conclusion, but not the problem's solution.

S2 did not record problem-solving information and inquiries. The subject could utilize math symbols while writing questions. The subject could prepare a completion strategy. S2 did not write down the reason, but the subject described it verbally. S2 could document the solution and completion phases during plan implementation. S2 could write mathematical solutions. S2 could translate the mathematical symbols into a problem situation but could not provide findings and arguments.

The Polya Stages guide S3 math problem-solving communication. S3 queries included math symbols. S3 could write a completion plan. S3 did not verbally explain the plan's purpose. S3 could not accurately express the solution and implementation reasons. S3 did not change the problem's mathematical symbols throughout re-examination. S3 lacked in terms of problem-solving, concluding and making arguments. S1 organized and strengthened mathematical concepts through discussion to understand the problem. S1 and S2

could regulate and strengthen mathematical reasoning through communication during planning, whereas S3 lacked these markers. S1, S2, and S3 could communicate mathematical concepts using mathematical language. S1 and S2 were able to explain their mathematical reasoning. S1 and S2 could communicate math concepts using mathematical language. S1, S2, and S3 could not analyze and assess others' math strategies and ideas. Only S1 can explain mathematical concept to the teacher, during reexamination.

For teachers, this research can be considered as a reference in observing students' mathematical communication processes. The research focuses on students' mathematical communication processes in writing and is supported by interviews, thus there are still opportunities for research on students' mathematical communication that is focused on different things, such as listening communication, representational communication and student discussion communication.

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