ERROR ANALYSIS OF LEARNERS' PROBLEM-SOLVING ABILITIES IN MATHEMATICS COURSES: A NEWMAN ERROR ANALYSIS (NEA) APPROACH

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Error analysis Problem solving Newman Error Analysis This study endeavors to systematically discern the spectrum of learner errors manifesting in problem-solving processes within the domain of mathematics, specifically focusing on the intricacies of integration, utilizing Newman Error Analysis (NEA) as the guiding framework. The cohort under investigation comprises 14 first-year students hailing from the Department of Physics Education at Musamus University, Merauke. Employing a descriptive methodology underscored by a qualitative orientation, this research incorporates tests and interviews as primary instruments for data collection. The tests are designed to meticulously evaluate the array of errors committed by learners, as per the tenets of Newman's model. Additionally, a subset of four participants is selected for in-depth interviews aimed at elucidating the underlying challenges encountered during problem-solving endeavors, which ultimately contribute to the genesis of learner errors. Analysis of the findings underscores the prevalence of comprehension errors, transformation errors, process errors, and encoding errors amongst learners grappling with problems centered on the subtopic of integration.

ABSTRACT

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INTRODUCTION

Mathematics, renowned for its pervasive influence, assumes a pivotal role across various domains encompassing science, technology, and everyday applications (Rohmah & Sutiarso, 2018). Particularly, its mastery lays the groundwork for subsequent academic pursuits, notably within natural sciences, engineering, and education (Tasman & Ahmad, 2018; Ayebo et al., 2017; Boz & Adnan, 2017; Rasmussen et al., 2014; Ellis et al., 2021). A proficient understanding of mathematics is paramount, given its fundamental significance in shaping the educational trajectory of learners. Indeed, inadequacies in grasping mathematical concepts can precipitate challenges in navigating advanced coursework. Reflecting its indispensability, mathematics assumes the status of a compulsory subject in educational curricula, as observed in Indonesia (Tasman & Ahmad, 2018). At Musamus University, Merauke, the inaugural year students in the Department of Physics Education are introduced to mathematics as an essential component of their academic journey. Notably, integration emerges as a focal point within the curriculum, showcasing its relevance and centrality (Bressoud, 1992). However, empirical evidence suggests that learners encounter obstacles in assimilating the concept of integration (Zakaria & Salleh, 2015), often resorting to rote algorithms devoid of deeper conceptual understanding (Dawkins & Epperson, 2014).

The challenge persists among first-year students enrolled in the Department of Physics Education at Musamus University, Merauke. Findings from our investigation reveal that a significant proportion, accounting for 35.7% of learners undertaking mathematics courses, have faced failure in the subject. Additionally, a notable subset achieves only marginal success, a circumstance with potential ramifications for their subsequent academic endeavors. Notably, a primary contributing factor to this trend is the deficiency in comprehending the concept of integration. This deficiency stems from a prevailing tendency among learners to prioritize procedural fluency over conceptual understanding (Tasman & Ahmad, 2018). Mathematics, in such instances, is often perceived merely as a series of algorithmic steps, devoid of deeper contextual understanding and significance.

The imperative for learners lies in the application of their acquired knowledge and the synthesis of previously learned concepts to arrive at solutions (Dawkins & Epperson, 2014). Indeed, the adeptness of learners in resolving mathematical problems holds profound significance, given the pervasive presence of mathematical challenges in future contexts (Dawkins & Epperson, 2014). However, persistent deficiencies persist among learners in this regard, as underscored by Güner and Erbay (2021). Notably, challenges manifest in the selection of appropriate problem-solving approaches, indicative of a broader need for enhanced strategic competence (Güner

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& Erbay, 2021). Recognizing the pivotal role of problem-solving strategies, particularly in nurturing learning motivation, underscores their indispensability in the educational landscape (Schukajlow et al., 2022).

The errors encountered by learners in problem-solving tasks STEM from a multitude of factors, encompassing intelligence levels and entrenched problem-solving habits. These errors exhibit a diverse spectrum of severity and complexity, warranting a systematic approach for their analysis to facilitate the formulation of effective interventions (Nuryati et al., 2022). Newman Error Analysis (NEA) method stands out as a comprehensive framework for categorizing error types within problem-solving contexts. In this investigation, we employed NEA to scrutinize learner errors in problem-solving. This methodology offers a nuanced understanding of the underlying factors contributing to learner errors, thereby informing targeted pedagogical strategies and interventions aimed at enhancing problem-solving proficiency (Siskawati et al., 2021; Ningrum et al., 2019; Son et al., 2019; Sumule et al., 2018; Suradi & Djam'an, 2021).

The stages of Newman's procedure, encompassing reading errors, comprehension errors, transformation errors, processing skill errors, and coding errors, delineate a structured framework for dissecting learner difficulties in problem-solving (Siskawati et al., 2021). Through the approach of NEA, this methodology serves as a diagnostic tool to evaluate the depth of learners' cognitive processes (Siskawati et al., 2021). Errors identified through NEA can be classified into cognitive and non-cognitive factors, with distinct problem-solving approaches delineated for each category (Nuryati et al., 2022; Sukoriyanto, 2020; Yunus et al., 2019).

Specifically, the factors contributing to errors can be segmented as follows: 1) Reading errors STEM from a lack of comprehension of the passage and the meaning of words in the question; 2) Misunderstanding arises from a deficiency in grasping the problem's context, hindering the identification of relevant information and query; 3) Transformation errors occur when learners struggle to translate the problem into mathematical expressions; 4) Processing skill errors manifest as difficulties in executing numerical operations due to a lack of procedural understanding; 5) Coding errors emerge when learners fail to accurately articulate the final solution, often attributed to overlooking reference materials or models during problem-solving (Nuryati et al., 2022; Sukoriyanto, 2020; Yunus et al., 2019). Against this backdrop, the objective of our study is to employ NEA to discern and characterize learner errors in problem-solving, particularly focusing on the subtopic of integration within the mathematics curriculum. Through this analytical approach, we aim to unravel the intricacies of learner challenges and devise targeted interventions to bolster problem-solving proficiency in mathematical contexts.

METHOD

This study adopted a descriptive methodology with a qualitative orientation, geared towards scrutinizing learner errors in the domain of mathematics, specifically during the resolution of integral problems, leveraging Newman Error Analysis (NEA). The cohort under scrutiny comprised 14 first-year students enrolled in the Department of Physics Education at Musamus University, Merauke. Employing essay-based test questions and interviews as principal data-gathering instruments, the research sought to delve into the nuances of learner errors. The test questions were meticulously crafted to gauge errors through the lens of Newman's framework. Furthermore, a subset of four participants was meticulously selected for interviews, aimed at dissecting the intricacies of the challenges they encountered while grappling with the posed questions, thereby shedding light on the genesis of their errors.

RESULTS

The analysis of students' responses to the test questions was conducted through the approach of Newman's indicators, aiming to pinpoint the specific areas of error occurrence across different dimensions of understanding. Subsequently, four participants were purposefully chosen to corroborate the challenges observed. These participants encompassed two individuals, labeled L1 and L2, whose errors predominantly stemmed from deficiencies in instrumental understanding, alongside two others, denoted as L3 and L4, whose errors were primarily rooted in relational understanding. Below, the findings derived from the participants' responses will be expounded upon.

Subject L1

The ensuing excerpt emanates from an interview session with participant L1. The outcomes of the examination conducted on Subject L1 are depicted in Figure 1.

- R : Can you elucidate the nature of the inquiry presented in the problem?
- L1 : The problem entails the task of integrating the provided function
- R : Were you able to perform the integration?
- L1 : In my understanding, I have indeed executed the integration as required
- R : Do you possess confidence in your solution?
- L1 Certainly. Just a moment... Integrating, correct? Oh, but it seems I inadvertently proceeded with differentiation instead. Ah, that's an oversight, isn't it?



Figure 1. Error by L1 in answering the question.



Figure 2. Error by L2 in answering the question.

Subject L2

The ensuing excerpt emanates from an interview session with participant L2. The outcomes of the examination conducted on Subject L2 are depicted in Figure 2.

- : Can you explain the nature of the inquiry presented in the problem? R
- L2 : The problem tasked me with integrating the expression $(x^2 + 4)^{10} x dx$
- R

R

: How did you approach solving the problem? : Initially, I substituted $(x^2 + 4)^{10}$ for x and du = 2x proceeded with the integration process $\frac{1}{2} \int u^{10} du$. L2

Consequently, I arrived at the solution $\frac{1}{2} \left(\frac{n^{11}}{11} + C \right)$. Hence, the final result is $\frac{(x^2 + 4)^{11}}{11} + K$ Are you confident in your answer?

Absolutely. I do apologize for the oversight in neglecting to multiply by a half. The correct outcome should L2 indeed be $\frac{(x^2+4)^{11}}{11} + K$.

Subject L3

The ensuing excerpt emanates from an interview session with participant L3. The outcomes of the examination conducted on Subject L3 are depicted in Figure 3.

- : Can you explain the objective of the problem? R
- L3 : The problem tasks us with determining the velocity after 2 seconds
- R : How did you approach solving the problem?
- Initially, I integrated the acceleration to derive the velocity, yielding the expression $\frac{dv}{dt} = (2t+3)^{-3}$. Subsequently, L3 : upon further computation $v = \int (2t+2)^{-3} 2 dt$, the result is obtained $v = -\frac{1}{4(2t+3)^2} + C$.
- : What are the subsequent steps? R
- Given the additional information that at a velocity of 4 m/s, the time taken is zero, we can deduce the constant's L3 value by substituting these values into the equation $v = -\frac{1}{4(2\ell+3)^2} + C$. Consequently, upon finding the constant

to be zero (C = 0), the velocity magnitude at 2 seconds is determined to be $v = -\frac{1}{4(49)}$

- R Are you confident in your answer? :
- L3 : Absolutely. It's concluded.

Subject L4

The ensuing excerpt emanates from an interview session with participant L4. The outcomes of the examination conducted on Subject L4 are depicted in Figure 4.

- : Can you elaborate on the task presented in the problem? R
- : The problem prompts us to illustrate that an object propelled upwards from the Earth with a specified initial L4 velocity will not return to the Earth
- : How did you approach addressing the problem? R

: Initially, I applied Newton's Second Law, $F = m \cdot a$, to the problem. Subsequently, I sought to determine the L4 velocity magnitude by differentiating the equation $F = m \cdot a$, resulting in $F = m \frac{dv}{dt}$. This led to the solution

$$m\frac{dv}{dt} = -mg\frac{R^2}{s^2}$$



Figure 3. Error by L3 in answering the question.



Figure 4. Error by L4 in answering the question.

- R : What subsequent steps did you take?
- L4 : Following variable separation, I arrived at $v \, dv = -g \, R^2 s^2 ds \leftrightarrow \int v \, dv = -g \, R^2 \int s^2 \, ds$. Upon integration, I obtained $\frac{v^2}{2} = \frac{g \, R^2}{s} + C$. Subsequently, when $v = v_0$ and s = R, I determined the constant's value, $C = \frac{1}{2} v_0^2 + g R$, resulting in $v^2 = \frac{2g \, R^2}{s} + v_0^2 + 2g R$.
- R : Are you confident in your answer?
- L4 : I must admit, I harbor some reservations regarding my response.

DISCUSSION

Based on the findings derived from both empirical analysis and interview assessments, subject L1 exhibited several noteworthy errors throughout their problem-solving process. Firstly, a comprehension error transpired wherein subject L1 misinterpreted the task's instructions, erroneously differentiating instead of integrating the problem. This lapse stemmed from a fundamental challenge in apprehending the problem's intended objective, indicative of broader difficulties among learners in grasping mathematical concepts, as highlighted in previous studies (Nuryati et al., 2022; Sukoriyanto, 2020; Yunus et al., 2019). Secondly, a transformation error ensued as subject L1 inaccurately transformed the problem into a differential format instead of integrating it as directed. This misstep underscores the learner's struggle in discerning appropriate problem-solving pathways, as noted in the literature (Utami et al., 2021). Thirdly, a process skill error occurred whereby Subject L1 incorrectly proceeded with differentiation instead of integration, indicative

of limited mathematical acumen in determining the requisite problem-solving steps (Utami et al., 2021). Lastly, an encoding error emerged in the form of an erroneous final answer, attributable to subject L1's misconception of the problem's requirements. Instead of computing the integral as instructed, the learner inadvertently pursued differentiation, resulting in a discrepancy between the obtained solution and the problem's stipulated task. This encoding lapse reflects the learner's challenge in articulating conceptual understanding into precise problem resolutions, exacerbated by earlier difficulties encountered in the problem-solving process (Utami et al., 2021).

Following the analysis and interview findings, subject L2 exhibited two errors. First, in the transformation process, specifically in the multiplication operation $\frac{1}{2} \left(\frac{u^{11}}{11} + C \right)$. Subject L2 failed to execute the necessary multiplication step $\left(\frac{1}{2} \right)$, as admitted during the interview, attributing the mistake to a difficulty in determining subsequent problem-solving steps (Utami et al., 2021). Second, an encoding error surfaced during the final answer composition by subject L2, stemming from a prior transformation misstep. This led to an erroneous outcome in the problem-solving operation. The encoding error highlights the learner's struggle in grasping conceptual nuances to derive accurate final conclusions, exacerbated by challenges encountered in preceding stages (Utami et al., 2021).

Based on the analysis of both the work and interview outcomes, subject L3 encountered a transformation error during problemsolving. This lapse occurred when subject L3 erroneously derived a constant value of zero by substituting v = 4 m/s and t = 0 into the second term of the equation $v = -\frac{1}{4(2t+3)^2} + C$, the value of the constant was obtained $C = \frac{145}{36}$. This instance exemplifies the challenges learners face in discerning subsequent steps required for problem resolution, as outlined in prior research (Utami et al., 2021). Furthermore, this transformation error led to an encoding error manifested in subject L3's final answer. This encoding error stemmed from previous errors, notably the misstep in the transformation operation, resulting in an erroneous outcome during the problem-solving process. The inability to articulate concepts accurately to derive the correct final result and conclusion reflects the learner's struggle, which is compounded by difficulties encountered in earlier stages (Utami et al., 2021).

Similarly, Subject L4 encountered a transformation error by conducting calculations on $v \, dv = -g R^2 s^2 ds \Leftrightarrow \int v \, dv = -g R^2 \int s^2 ds$ and obtained the result $\frac{v^2}{2} = \frac{g R^2}{s} + C$. When $v = v_0$ and s = R, the constant $C = \frac{1}{2}v_0^2 + g R$, resulting in $v^2 = \frac{2g R^2}{s} + v_0^2 + 2g R$. During the process of deriving constants, a transformation error occurred, yielding an erroneous outcome of $C = \frac{1}{2}v_0^2 + g R$. This phenomenon aligns with existing research elucidating that transformation errors often stem from learners encountering challenges in discerning subsequent steps required for problem resolution (Utami et al., 2021). Such discrepancies not only impede the accurate determination of constants but also exert cascading effects on the formulation of final results, precipitating encoding errors. Encoding errors, characterized by inaccuracies in articulating concepts to attain precise conclusions, represent a manifestation of learners grappling with the conceptual intricacies necessary for problem-solving (Utami et al., 2021). These challenges are invariably linked to preceding stages of comprehension and transformation within the problem-solving process.

CONCLUSION

Based on empirical investigations conducted among first-year students enrolled in the Physics Education Department at Musamus Merauke University, with a focus on their performance in the mathematics curriculum, particularly within the domain of calculus, the findings can be succinctly delineated as follows: learners exhibited various types of errors, including comprehension lapses, transformation challenges, procedural inaccuracies in skill execution, and encoding discrepancies in their responses. Comprehension errors were observed stemming from students' struggles in grasping problem contexts, consequently affecting their ability to apply appropriate methodologies, devise problem-solving strategies, and accurately articulate solutions. Transformation errors, entailing difficulties in comprehension, application, and problem-solving, were notably prevalent, notably manifesting as challenges in interpreting problem scenarios, identifying pertinent data, applying relevant equations, and articulating coherent methodologies for obtaining precise solutions. Process errors and subsequent inaccuracies in final outcomes were attributed to students' struggles in applying established standards and techniques, thereby impeding their capacity to arrive at correct answers. Consequently, this underscores the necessity for further research endeavors aimed at enhancing these specific cognitive competencies among students.

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AUTHOR CONTRIBUTIONS

AE was involved in conceptualizing the study, designing the methodology, conducting formal analysis, and drafting the original manuscript, editing the manuscript, and managing project administration. DJP contributed to validation processes, and participated in reviewing and editing the manuscript.

CONFLICT OF INTEREST STATEMENT

Regarding the research, writing, and publication of this paper, the authors declare no conflict of interests.

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