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FOCUS ON THE CORRECTNESS OF RESULTS: FACTORS CONTRIBUTING TO INVISIBLE THINKING PROCESSES IN HIGH-RESILIENCE MATHEMATICAL STUDENTS

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ABSTRACT

The cognitive process of thinking plays a pivotal role in understanding one's surroundings and effectively solving problems. This research endeavors to delve into the intricate factors contributing to the concealment of thinking processes in writing among students with high mathematical resilience. Employing a qualitative approach, specifically adopting a case study research design, the study concentrates on the students enrolled at SMPI Al-Lailiyah Sumenep. The comprehensive dataset is derived from the results of numeracy-type tests and interviews conducted with the student. The outcomes of this research shed light on a persistent challenge observed during the problem-solving phase among students exhibiting high mathematical resilience. A predominant tendency is noted, wherein these students habitually prioritize the correctness of final results over conveying the step-by-step reasoning behind their solutions. Several causative factors contribute to this phenomenon. First, the students' established approach to problem-solving, emphasizing the end result, impedes the transparent expression of their cognitive processes in writing. Additionally, past experiences related to the types of questions posed, the depth of their understanding of mathematical concepts, a lack of feedback from teachers, and the absence of visible thinking routines in written assignments during the learning process collectively contribute to the challenge.

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INTRODUCTION

The essential competencies required for individuals in future societies encompass three key aspects: thinking skills, collaborative abilities, and proficiency in solving complex problems (Kim & Lim, 2019). In this context, thinking is defined as a cognitive process that plays a crucial role in understanding one's surroundings (Lim, 2022). Thinking ability refers to an individual's skill in comprehending tasks and validating the given responsibilities (Czocher, 2016). The importance of robust thinking skills becomes evident in the thorough understanding of encountered problems (Ferrini-Mundy, 2000), as cognitive activities naturally unfold when faced with challenges (Hanney, 2018). Therefore, the mathematical thinking abilities of students can be elucidated as their capability to grasp mathematical tasks, navigate through problem-solving procedures, comprehend underlying concepts, and draw meaningful conclusions from educational activities (Supriadi et al., 2015). These competencies collectively form a foundation for individuals to thrive in evolving societal contexts.

Various studies have extensively investigated the cognitive facets associated with mathematical thinking abilities. Albay and Eisma (2021) posit that students who manifest superior performance in task execution concurrently exhibit refined mathematical thinking skills. The maturation of these cognitive skills is intricately linked to students' prior exposure to expansive and meaningful learning experiences, which significantly contributes to the augmentation of their subsequent mathematical thinking prowess (Möhring et al., 2021). The optimal efficacy of the learning process, specifically concerning the cultivation of mathematical thinking skills within the classroom milieu, is realized when complemented by the facilitative role of an adept teacher capable of rendering students' cognitive processes perceptible and observable (Fraser et al., 2019).

The success of the learning process is influenced by various factors, among which the psychological resilience of students plays a crucial role, encompassing traits such as perseverance and the ability to confront challenges, collectively referred to as mathematical resilience (MR) (Glenn et al., 2018; Johnston-Wilder & Lee, 2010; Lutovac, 2019). MR has been observed to positively impact diverse mathematical abilities (Komala, 2018). Preliminary investigations conducted at SMPI Al-Lailiyah center around the theory of problem-solving framed within the context of students' levels of MR, revealing that "students with elevated MR outperform their peers in mathematical tasks compared to those with moderate or low MR levels" (Attami et al., 2020; Rahmatiya & Miatun, 2020).

High MR equips students with enhanced capabilities to navigate challenges and obstacles encountered in the process of learning mathematics, as indicated by prior research (Johnston-Wilder & Lee, 2010; Komala, 2018; Kooken et al., 2016; Li et al., 2019; Musich et al., 2022; Yeager & Dweck, 2012). This theoretical framework is pertinent to the present study, asserting that "students with heightened MR exhibit slightly superior proficiency in solving mathematical problems in comparison to their counterparts with moderate or low MR levels". The numerical questions provided to students align with the genre of questions employed in the Minimum Competency Assessment (AKM) at the junior high school level. These numeracy-type questions, derived from benchmarks such as the Programme for International Student Assessment (PISA) and Trends in International Mathematics and Science Study (TIMSS), aim to transition students from mere rote memorization to analytical thinking based on information (Pusat Asesmen dan Pembelajaran, 2020).

Building on the various studies, this research will delve further into an exploration aimed at unraveling the factors that contribute to the phenomenon of students with high MR struggling to articulate their thoughts in written responses. The primary objective of this study is to elucidate the impediments faced by students with high MR in expressing their cognitive processes through written answers. The anticipated outcome of this research is to empower teachers with insights for targeted interventions, facilitate an understanding of students' evolving mathematical comprehension, identify effective teaching methodologies to enhance mathematical thinking activities, streamline assessment processes, and pinpoint areas for improvement in the overall learning trajectory. Explicit articulation of students' thoughts fosters crucial skills in mathematics, encompassing idea formulation, expansion, and refinement through collaborative sharing (Hull et al., 2011). Comprehending these factors assumes paramount importance in crafting interventions designed to enhance students' proficiency in articulating their mathematical thinking through written expression. Such interventions necessitate a holistic approach, encompassing both mathematical and communicative dimensions. This involves cultivating a supportive learning environment that encourages students to explore and articulate their mathematical thoughts effectively. Additionally, providing structured opportunities for students to develop reflective writing skills within the context of mathematics is integral to fostering their overall ability to convey complex mathematical ideas in written form.

METHOD

This research adopts a qualitative approach utilizing a case study research design to delve deeply into specific incidents, processes, or activities involving one or more individuals (Creswell, 2009). Data for the study are sourced from both test results and interviews. The research was conducted on May 24, 2022, at SMPI Al-Lailiyah Sumenep, Indonesia. The primary data source is a student at SMPI Al-Lailiyah Sumenep with high mathematical resilience who has encountered challenges in articulating their thought processes in writing. The selected student, adept at verbal expression, facilitates data collection during the interview phase of the research. The employed data collection techniques encompass tests and interviews. The data analysis technique adheres to the Miles and Huberman approach (Sugiyono, 2016), involving (1) data collection, (2) data reduction, (3) data presentation, and (4) drawing conclusions. The indicators employed to assess students' thinking processes in this study align with Mason's thinking theory (Mason et al., 2010; Wardhani et al., 2016), as delineated in the Table 1.

Table 1. Thinking process indicators.

Stage	Aspect	Indicator
Entry	Know	1. Write down the results of problem identification
	Want	2. Group and sort information
	Introduce	3. Write down the results of problem identification and grouping information into mathematical symbols
Attack	Try	1. Propose/write a problem solving plan (strategy)
	Maybe	2. Test the plan and write down detailed completion steps
	Why	3. Present systematic completion steps, by writing down the reasons for each completion step, so that other people can see the truth of the process carried out
Review	Check	1. Checking the completion steps such as accuracy of calculations, reasons, and suitability of the completion steps to the question
	Reflect	2. Reflect on problem solving steps
	Extend	3. Look for other solutions

RESULTS

Preliminary Investigation

In the assessment of mathematical resilience (MR) within the student cohort, the researcher employed a MR questionnaire comprising three pivotal indicators. Firstly, the Value indicator gauged students' perceptions regarding the intrinsic significance of learning mathematics for both current and future accomplishments. The second indicator, Struggle, elucidated a positive disposition where students exhibited resilience by not easily succumbing to challenges or failures during the learning process, leveraging such experiences to foster self-motivation. The third indicator, Growth, encapsulated the belief that existing mathematical knowledge could be further enhanced (Kooken et al., 2016). The questionnaire employed in this study underwent validation by expert assessors. The classification of mathematical resilience levels adhered to the taxonomy established by Fatimah and Fitriani (2021) and Kurnia et al. (2018). During the initial investigation, it was determined that 13 students exhibited low levels of MR, while 3 students demonstrated moderate MR levels, and 8 students showcased high MR levels. Figure 1 provides a representative analysis of the gathered responses.

Analysis of students' responses in the preliminary study reveals a shared challenge among individuals across varying levels of MR, a notable struggle to articulate their cognitive processes in writing when tackling problems. This is evident in the manner students approach problem-solving without furnishing written explanations or a systematic breakdown of their solutions on answer sheets. Even among those who may lack proficiency in expressing solutions in writing, foundational skills such as observation, deduction, induction, collaborative reasoning, strategy formulation, decision-making, interaction with peers, and articulation of observations related to specific cases may still be apparent. However, this highlights a persistent difficulty in effectively conveying their thoughts (Hull et al., 2011; Novitasari et al., 2020; Salmon, 2016).

In this context, the concept of visible thinking, characterized by the clarity and transparency of an individual's cognitive processes, assumes paramount importance (Hull et al., 2011). The challenge lies in students providing comprehensive and accurate written accounts of their observations pertaining to a problem. Difficulties may arise in discerning patterns within problems, particularly in the utilization of symbols, numerics, and mathematical operations (Alexander et al., 2020). Nevertheless, written assignments, despite these challenges, present unique advantages due to their ease of visibility and analysis, routine feasibility, and seamless integration with mathematical content, as compared to other modes of academic work (As'ari et al., 2019).

The Cognitive Processes of High-Resilience Mathematical Students Do Not Manifest Prominently When Engaging in Tasks Pertaining to Numeracy

Several factors contribute to the continued challenge of students with high mathematical resilience in expressing their thinking processes. The identification of these factors was undertaken through the administration of a test containing numeracy-type problems of diverse cognitive levels. The data were further enriched by interview results to provide a comprehensive understanding. The students engaged with three numeracy-type problems (Figure 2), covering cognitive levels involving both application and reasoning.

Throughout the problem-solving process, the researcher meticulously observed the behaviors exhibited by the students. This combined approach, integrating quantitative data from the test and qualitative insights from interviews, allows for a thorough exploration of the multifaceted challenges faced by students with high mathematical resilience in articulating their cognitive processes.

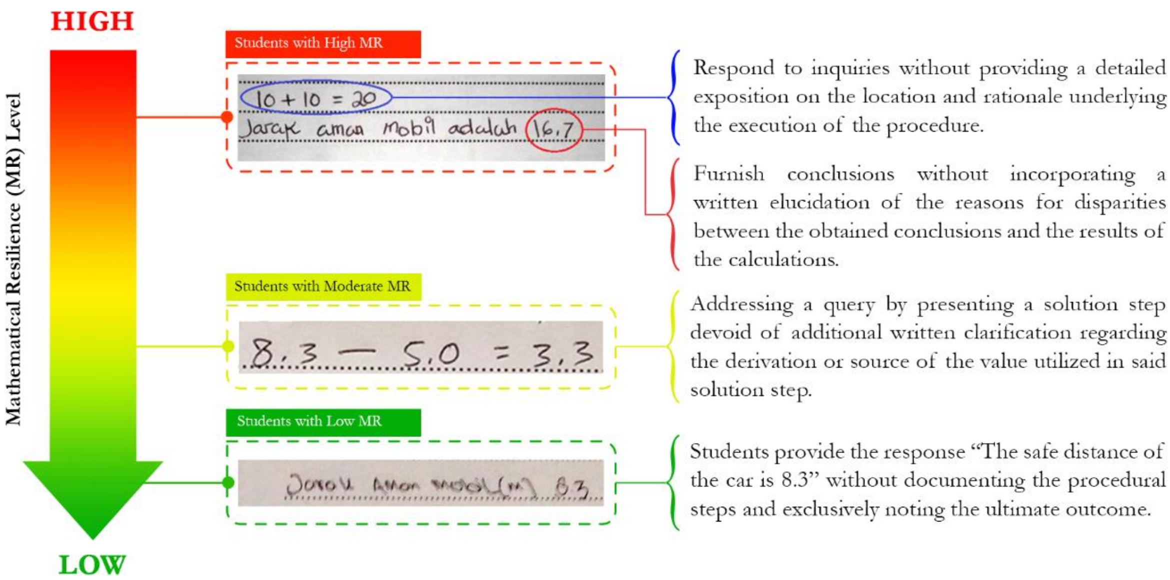
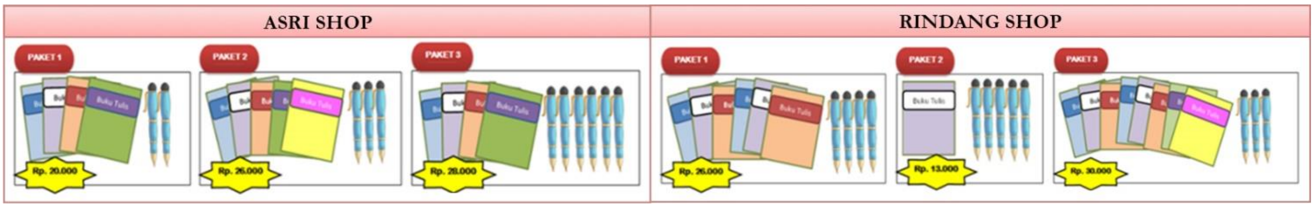


Figure 1. Preliminary investigation results.

1. During school holidays, Rini intends to purchase stationery in the form of notebooks and pens. Rini observed various packages of stationery with the same brand available at both the Asri shop and the Rindang shop, outlined as follows.



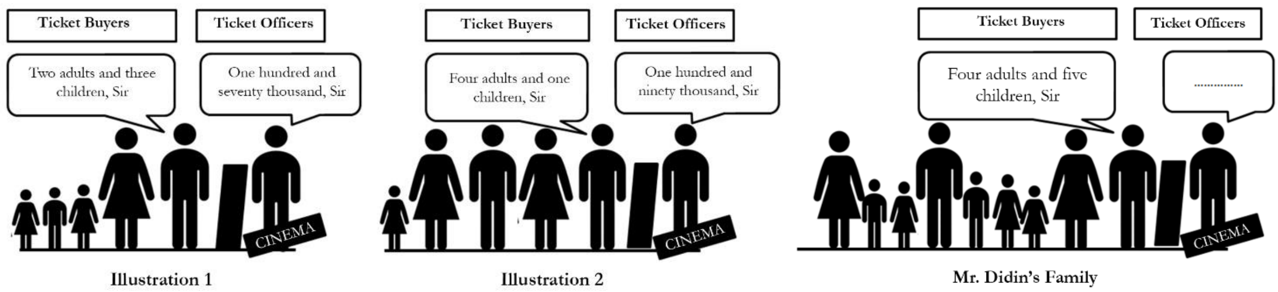
In consideration of the visual representation provided above, kindly respond to the following inquiries:
a) Ascertain the cost of a singular notebook and pen at both the Asri shop and the Rindang shop.
b) Drawing from the acquired information, it has come to light that the Asri shop offers a 25% discount for a minimum expenditure of IDR 50,000. Given Rini's intention to procure 8 notebooks and 5 pens, kindly opine on the most judicious course of action to enable Rini to acquire a stationery package meeting her requisites while minimizing her expenditure. Specify the preferred store and the recommended package.

2. In commemoration of the Republic of Indonesia's Independence Day, RT administrators organized two competitions, namely the Cracker Eating Competition and the Searching for Coins in Flour competition. Each participant is eligible for a cash prize, the quantum of which is contingent upon both the total quantity of crackers consumed and the number of coins unearthed. The ensuing table presents an illustrative simulation delineating the monetary rewards accrued by two participants, predicated on their respective tallies of crackers ingested and coins discovered.

Name	Result of Competition		Prizes
Sisi			Rp 12.000
Ali			Rp 12.000

Considering the data presented in the aforementioned simulation table, respond to the ensuing inquiries:
a) Given that Roki consumed 4 crackers and discovered 5 coins, and Lala consumed 3 crackers while finding 6 coins, calculate the disparity in the monetary prizes awarded to Roki and Lala.
b) Kiki's participation in the competition resulted in a prize of Rp. 9,000. The plausible combination of the number of crackers consumed by Kiki and the coins acquired could be deduced through further examination.

3. On weekends, Mr. Didin intends to take his family to the cinema. Upon arrival, he observed several adults and children waiting in line to purchase tickets. The pricing for tickets, applicable to both adults and children, is elucidated in the ensuing conversation as depicted in the two accompanying picture illustrations.



a) Drawing upon the illustration of Mr. Didin's family, the ticket fee levied by the ticket sales officer for a family composition mirroring that depicted in the illustration is...
b) The ticket sales officer conveyed that a 15% discount would be applicable to the ticket price for a group of 5 children. Subsequently, having initially provided payment, Mr. Didin received change in the amount of...

Figure 2. Numeracy type questions.

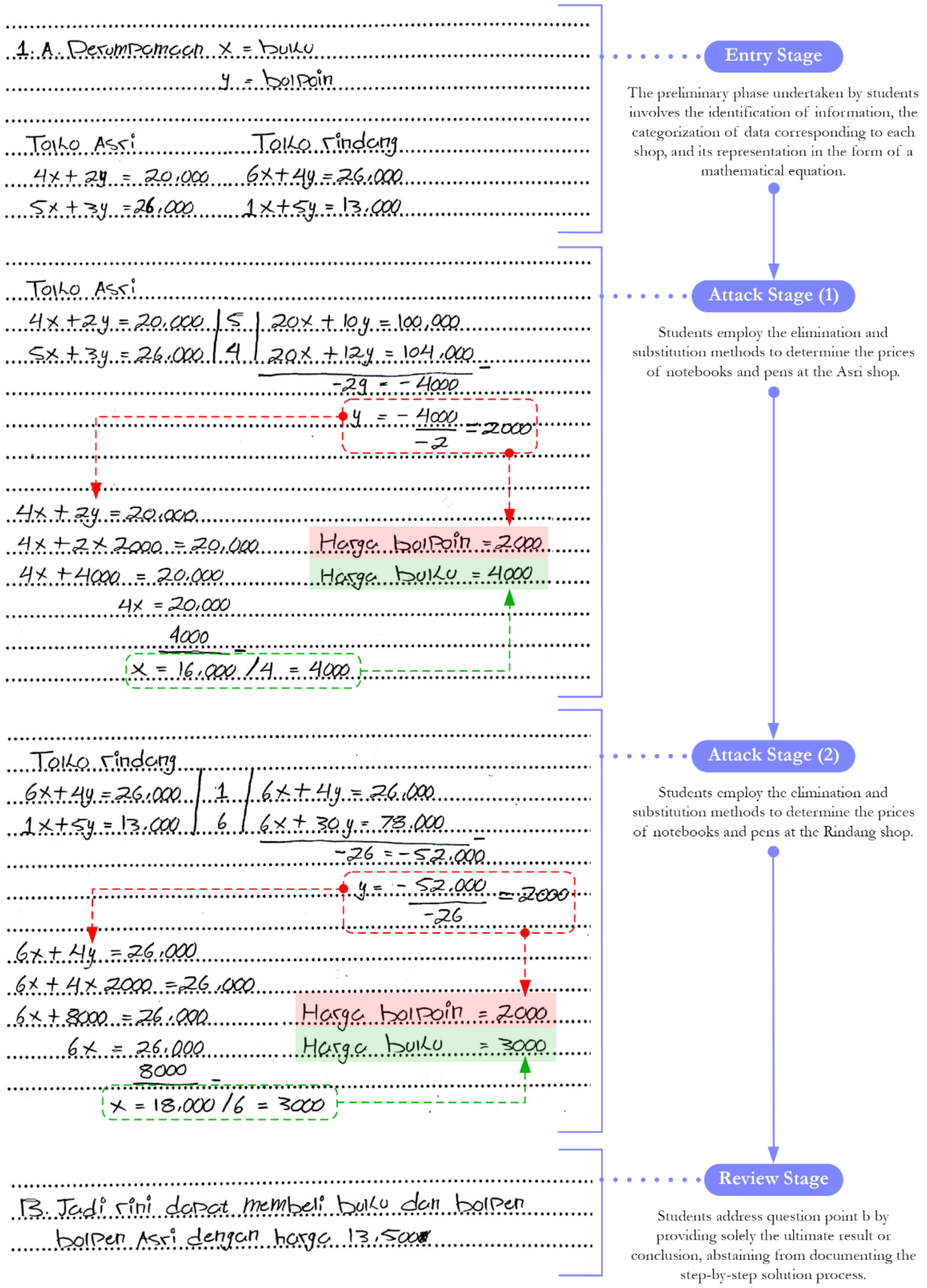


Figure 3. Student's written answer to the first question.

In the process of addressing the first problem, the student commenced by thoroughly reading the given question and proceeded to document the identified problem on the answer sheet (Figure 3). However, upon tackling part a of the question, the student omitted crucial information, such as specifying what is known and what is being sought in the problem. Instead, the student directly assigned values to the variables, for instance, x = book and y = pen. This indicates a potential gap in the student's understanding of the concept of variables. The student seems to perceive variables as symbols representing a predetermined type rather than symbols signifying a value based on a specific type. Consequently, the variable is construed as an entity possessing an inherent value or quantity. In light of this, employing the equal sign (=) with a particular type of object to represent a variable is deemed inappropriate. A more accurate representation would involve stating, for instance, that x equals the price of one notebook, and likewise for y. Subsequent to this, the student proceeded to formulate equations based on the representation of the packages provided in the problem, corresponding to each store.

Contrastingly, when addressing part b of the problem, the student presented only the final solution without demonstrating the intermediate steps. As revealed in the conducted interview, at this stage, the student exhibited a robust understanding of the information in question 1, parts A and B. The research participant adeptly categorized and constructed a mathematical model in alignment with the information provided in the problem. Specifically, the student formulated the mathematical model based on the prices of each package according to their respective stores.

In the attack stage, particularly for part A of the question, the student adeptly formulated a solution strategy and successfully applied it to resolve the problem. The student arrived at the correct answer, yet abstained from furnishing explanations for each step undertaken. A similar pattern emerged when addressing part B of the problem, wherein the student not only omitted explanations for the steps but also exclusively presented the conclusion or final answer without delineating the solution process. In the interview, the student articulated a preference for providing concise answers, including only conclusions or final results on the answer sheet. This practice, as conveyed by the student, was ingrained through prior experiences in in-class problem-solving sessions. An excerpt from the interview with the student sheds light on this practice.

R : How did you determine that Rini could buy a book and a pen at the Asri store for Rp13,500? Can you elaborate on the process?
S : I considered the 25% discount at the Asri store for a minimum purchase of Rp50,000, which applied to packages 2 and 3. I added the prices of these packages, totaling Rp54,000. Multiplying this sum by 25%, I obtained the result of Rp 13,500. That's why I provided that answer.
R : Why did you choose not to document the steps or process of solving on your answer sheet, despite explaining it to me just now?
S : Typically, when there's a problem, I answer it in that manner. The crucial aspect is to arrive at the correct answer, and as long as I find it, I consider it sufficient.
R : Have you ever experimented with a different approach?
S : No, I haven't.

From the provided interview excerpt, it is evident that the student's approach to problem-solving centers on achieving the correct final result, leading to a perception that documenting the solution process holds lesser importance. Additionally, the student displays a limited understanding of the concept of a discount, as reflected in the interview snippet where the calculation process concludes by multiplying the original price of the selected item by the discount percentage, without deducting this value from the initial price. The combination of a less-than-robust grasp of mathematical concepts and the inclination to prioritize the accuracy of the final answer contributes to the student's challenge in expressing the thinking process in writing on the answer sheet. Another inhibiting factor is the absence of feedback from the teacher concerning the students' work, as indicated by the student's statement that their problem-solving approach aligns with what is typically done on the answer sheets provided during class.

During the review stage, the student engages in the process by iteratively examining and revisiting the answers without undertaking the substitution of values for one notebook and pen into the package equations corresponding to each store. As per the student's assertions, the employed problem-solving approach aligns with a methodology commonly instructed and acquired in the classroom setting. Notably, the student articulates a habitual omission of validation procedures, such as substituting the derived results into equations derived from known information, opting to conclude the problem-solving process once the answer is obtained. An excerpt from the conducted interview with the student provides insight into this approach.

R : Do you perform checks on the answers you provide? For example, checking the accuracy of calculations, the process, and the alignment of the answers with the questions.
S : Yes.
R : How do you verify the solution you obtained from question number 1?
S : I check it by reading and reviewing my answer.
R : Have you tried any other methods to solve question number 1?
S : No.
R : Why didn't you try?
S : Because the method I use is easy for me.

While tackling this question, the student exhibited precision in determining the prices of one notebook and one pen, illustrating engagement in the process of cross-verifying the answers. Furthermore, the student showcased the capacity to introspect on assumptions made during the problem-solving endeavor, delineating the various stages of the solution and pinpointing challenging elements within the problem.

Transitioning to the second question (Figure 4), during the entry phase of question 2a, the student manifested a commendable grasp of the information conveyed in the problem, effectively translating it into mathematical expressions. Consistent with the previous question, however, the student displayed a constrained understanding of the concept of variables.

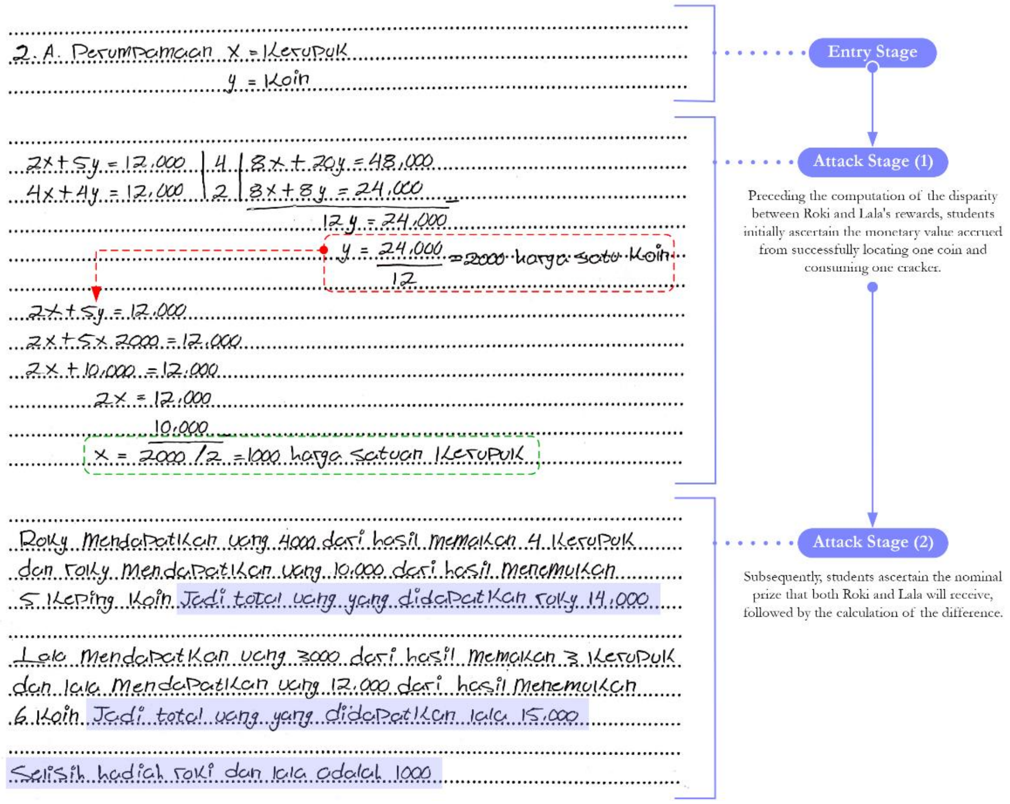


Figure 4. Student's written answer to the second question.

In the attack phase, the student exhibited adeptness in devising a thoughtful solution strategy and effectively applied it to unravel the problem. The student meticulously calculated the monetary value associated with one chip and one coin, simulated through the gifts received by Sisi and Ali. Subsequently, the student determined the disparity in gifts received by Roki and Lala. Regrettably, within this phase, the student abstained from furnishing a written explanation on the answer sheet elucidating the solution steps undertaken. In relation to question 2b, the student refrained from providing an answer, citing confusion in navigating the problem.

In concluding the second question, the student encountered challenges in articulating the thinking process in written form. Analysis of both the interview responses and the written test highlights an additional factor contributing to the opacity of the student's thinking process in writing – the influence of past experiences with particular question types. The student evidently feels more at ease when addressing questions at the applying level, as opposed to those at the reasoning level. Even when responding to applying-level questions, the student supplies partial solution processes, suggesting a reluctance to engage fully with reasoning-level questions. This implies that numeracy-type questions may not have been effectively incorporated into the learning process.

In the review phase, the student undertakes this process by revisiting and reading the answers, without engaging in the substitution of results obtained from the successful consumption of the chip and finding one into the equation derived from previously known information. Based on the student's statements, the employed problem-solving approach aligns with a methodology commonly instructed and learned in the classroom. The following excerpt from the interview with the student sheds light on this approach. The student explicitly states a customary avoidance of validation procedures, such as substituting derived results into equations formed from known information, preferring to conclude the problem-solving process upon successfully answering the question.

R : Do you perform checks on the answers you provide? For example, checking the accuracy of calculations, the process, and the alignment of the answers with the questions.
S : Yes.
R : How do you verify the solution you obtained from question number 2?
S : I check it by reading and reviewing my answer.
R : Have you tried any other methods to solve question number 2?
S : No.
R : Why didn't you try?
S : Because the method I use is easy for me.

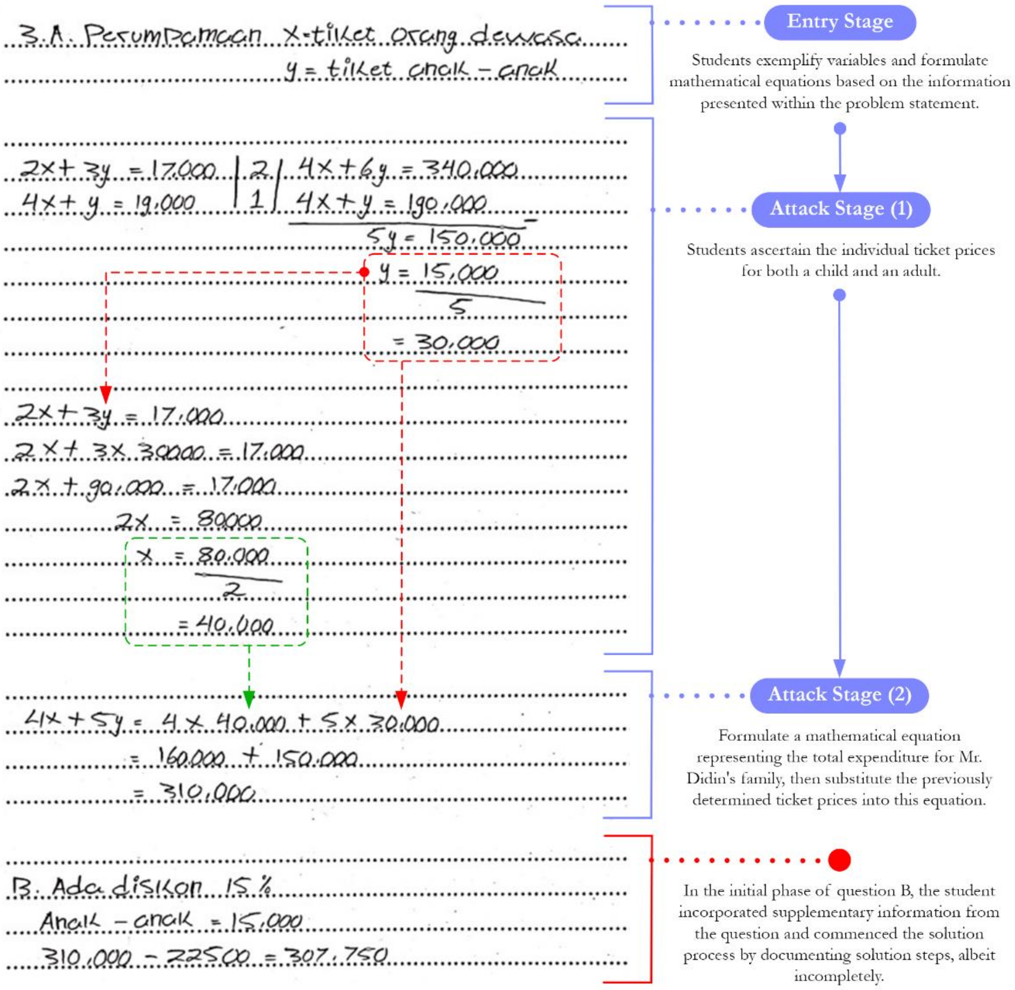


Figure 5. Student's written answer to the third question.

While addressing the third problem (Figure 5), denoted as question 3a, during the entry stage, the student demonstrated a proficient understanding of the information presented in the problem. However, akin to previous instances, in the process of formulating assumptions, the student exhibited a restricted grasp of the concept of variables. The student persisted in using the equal sign (=) for entities with a value or magnitude compared to those lacking a specific value or magnitude. Notably, at this stage, the student deviated from the practice of documenting the results of problem identification, such as the information queried and known in the problem. Instead, the student promptly proceeded to make assumptions about variables before embarking on problem-solving.

In responding to question 3b, the student appeared to incorporate some additional information gleaned from the problem. However, this data was presented without an accompanying explanation elucidating where and why certain information was utilized in the problem-solving process. Furthermore, the student adeptly translated the simulated images in the problem into mathematical equations, as illustrated in the provided image.

During the attack stage (problem-solving), the student navigated through the solution process based on the preconceived formula. According to the interview, the student indicated that prior to tackling the problem, they devised a plan encompassing the utilization of elimination and substitution processes to ascertain the ticket prices for one adult and one child. This strategic process was undertaken to facilitate the determination of the total amount in rupiahs that Mr. Didin would expend for the entire family's movie experience at the cinema. The formulated solution plan was effectively executed by applying it to address the problem. However, on the answer sheet, the student omitted comprehensive reasons or explanations for each step of the solution. Consequently, upon review, others might encounter ambiguity regarding the origin of those steps. The student clarified that they are accustomed to working on analogous problems without providing extensive explanations. Similarly, while resolving problem 3b, the student offered only a brief and incomplete depiction of the solution process. The following excerpt from the interview with the student provides further insight.

R : How did you go about solving problem number 3 point B?
S : For point B, I calculated the discount first, the ticket cost for Mr. Didin's child is Rp150,000, so Rp150,000 multiplied by 15%, the result is Rp22,500. So, Mr. Didin's refund is Rp307,750, obtained by subtracting Rp22,500 from Rp310,000.

R : Why didn't you write down the steps or solution process on your answer sheet as you explained to me earlier, especially for the answer to point B?
S : Usually, when there's a problem, I answer like that. The important thing is to have the answer.

The student's statement suggests an omission of the calculation process for the discount from the price and the subsequent 15% discount offered by the ticket booth attendant. At this juncture, a computational error occurred during the subtraction of Rp310,000 and Rp22,500. The student mistakenly arrived at Rp307,750 instead of the correct figure, Rp287,500.

In the review stage, as per the interview, the student re-evaluated the crafted solution, verifying its coherence with the question posed in the problem. The student performed this check without substituting the obtained result into the equation derived from the known information. The student acknowledged a customary avoidance of validation procedures, preferring to conclude the problem-solving process when they feel capable of answering the question. This tendency is evident in the test results, wherein the student accurately determined the amount Mr. Didin had to pay. The student exhibited the ability to reflect on potential solution pathways, recognizing the completion steps and challenging aspects of the question. The student admitted to scrutinizing the answer by inspecting and validating each step of the executed solution. However, it becomes apparent that the student remained oblivious to a calculation error made while working on problem 3b. The following excerpt from the interview with the student elucidates this oversight.

R : How do you verify the solution you obtained from question number 3?
S : I check it by reading and reviewing my answer.
R : Have you tried any other methods to solve question number 3?
S : No.
R : Why didn't you try?
S : Because the method I use is easy for me.

During the process of checking their answers, the student reviews their solution by observing and correcting it without conducting a thorough check by substituting the obtained results into the equation derived from the known information. The student has explicitly stated that they typically avoid such checks, including substituting results into equations from known information, and cease the validation process when they feel confident in their ability to answer the question.

Through an analysis of the written test results and interviews, it becomes evident that the student continues to struggle in manifesting their thought processes in writing on the answer sheet. This difficulty stems from the student's inclination to prioritize the correctness of results, past experiences related to specific problem types, a limited understanding of previous mathematical concepts, and a lack of comprehensive feedback from the teacher, particularly during the problem-solving process in the classroom. Building upon these factors, an additional identified issue is the absence of a routine promoting visible thinking in written tasks during the learning process. This is discernible from how students respond to written tasks, and the observed habits during problem-solving indicate a deficiency in implementing practices that encourage visible thinking in writing throughout the learning process. This is further corroborated by the dearth of feedback from teachers on the processes or steps of the student's solution when engaged in practice problems in the classroom.

DISCUSSION

Habits that Prioritize the Truth of the End Results

During the problem-solving phase (the attack phase), students characterized by high mathematical resilience (MR) encounter challenges in articulating their thought processes in writing on the answer sheet. Notably, they tend to skip providing detailed explanations for each step taken to solve the problems, often presenting only a final conclusion. This tendency is rooted in the habits cultivated during their learning experiences, resulting in missed opportunities for expressing and developing their thinking abilities. This observation aligns with assertion of Hwang et al. (2021), underscoring that habits and approaches ingrained during the learning process significantly influence success or failure in mastering mathematical concepts. Consequently, the classroom environment plays a pivotal role in shaping the foundational skills of students, with habits prioritizing correctness over the process potentially impeding the development of their thinking abilities. Such habits pose challenges for educators in accurately assessing students' work. Rosen and Tager (2014) also highlight that habits emphasizing a fixation on correctness or final results may indicate persistent obstacles in students' thinking processes. In light of these challenges, instilling habits that prioritize rendering students' thinking visible can prove instrumental in evaluating performance patterns and providing necessary scaffolding for their thinking processes. Leveraging tools such as concept mapping, designed to enhance thinking, can further offer support, guidance, and expansion of students' cognitive processes. Students exhibit effective thinking activities when they not only focus on generating computational answers and communicating results but also provide further interpretation or critical reflection on the problem or the given answer (van Dooren et al., 2010). In the context of this research, students actively engage in problem-solving by meticulously documenting the solution steps they deem crucial on the answer sheet. There exists a prevalent belief among students that the crux of problem-solving lies in successfully arriving at the answer. However, a common habit among students is to offer answers without due consideration for the underlying process of solution steps, such as the derivation of the utilized equation and the sequential steps involved. This habit results in students concentrating solely on the correctness of the results and the mere presence of the answer on their answer sheets, neglecting the rich contextual details of the problem-solving process.

Previous Experience Regarding the Type of Questions Given

Students have yet to manifest their cognitive processes effectively in written tasks, a tendency influenced by the nature of the presented questions. Students typically engage in tasks situated at the applying level, wherein the process predominantly entails the application of previously acquired concepts and formulas. This stands in contrast to numeracy-type questions at the reasoning level, necessitating students to adeptly employ their reasoning abilities and leverage a spectrum of information assimilated over time (Pusat Asesmen dan Pembelajaran, 2020). This observation aligns with the contention of Sheppard and Wieman (2020), positing a close interconnection between students' knowledge, characterized as their mathematical thinking, and elements such as their experiences (Røykenes, 2016), the diversity of task levels, and solution strategies acquired within the classroom context (Perry et al., 2022).

Stylianides and Stylianides (2022) underscore the imperative for teachers to tailor task designs in accordance with the unique needs of students. These needs extend beyond mere success in solving mathematical problems, encompassing requisites that can enrich students' mathematical skills in alignment with contemporary developments. Consequently, teachers must augment their proficiency in formulating and diversifying tasks at various cognitive levels. This strategic approach ensures the evolution of students' knowledge, fostering self-motivation and autonomy in tackling mathematical tasks and cultivating their mathematical abilities.

Understanding of Mathematical Concepts

One determinant contributing to the opacity of students' cognitive processes lies in their comprehension of previously learned mathematical concepts. This aligns seamlessly with van Garderen et al. (2020) assertion that a profound understanding of mathematical concepts enhances thinking processes, enabling the demonstration and application of ideas, ultimately leading to the adept resolution of mathematical problems. The establishment of an educational environment that nurtures students' comprehension abilities assumes paramount importance in the field of mathematics education (Gulkilik et al., 2020).

For students to articulate their understanding through specific pedagogical approaches, Legesse et al. (2020) underscore that, in the realm of mathematics learning, guiding students to exhibit their cognitive processes allows for the development of both conceptual and procedural understanding. The efficacy of this approach hinges on the extent to which students can provide explanations, justifications, formulate conjectures, pose questions, compare solution procedures, and engage in the reciprocal sharing of ideas.

Furthermore, in their roles as facilitators and classroom managers, teachers must possess the ability to empower students to articulate their thoughts effectively. This capability equips teachers to assess and strategically plan the subsequent steps in the learning process (Amador et al., 2022).

Teachers are Not Used to Giving Feedback

Teachers assume a pivotal role in shaping students' success in the learning process (Pitts et al., 2018). Therefore, another contributing factor to the challenge faced by students with high mathematical resilience in articulating their thought processes when addressing numeracy-type problems is intricately linked to the teacher's role in the learning journey. Building upon the previously discussed factors, students grapple with challenges in expressing their cognitive processes due to their predominant emphasis on the accuracy of the results during problem-solving.

Students report adhering to specific solution processes when tackling problems, aligning with the knowledge imparted in the classroom. They follow these prescribed steps, often forgoing exploration of alternative solution methods. This pattern suggests that the habit is reinforced by the feedback or guidance provided by teachers in evaluating the completeness of the employed solution processes. Consequently, students believe that the steps they undertake are inherently correct. Some students even articulate that achieving the correct answer is paramount, signifying the attainment of the desired outcome. This aligns with findings from research by Brooks et al. (2021) and Burgess et al. (2022), underscoring that assessments and feedback from teachers can enhance students' confidence in their mathematical abilities. Furthermore, such feedback aids in error recognition, prompting active engagement in seeking, evaluating, and improving for subsequent learning, thereby averting the repetition of the same mistakes. Vries et al. (2022) corroborate these findings by highlighting that through assessment and feedback, teachers can make informed decisions and chart a more effective learning trajectory for students.

Lee and Simpkins (2021) underscore that students' achievements are significantly shaped by the support and role of teachers. The instructional content and guidance provided in the classroom are instrumental in molding students' abilities and skills. Consequently, students rely on teachers as both supporters and guides during the learning and problem-solving activities (Forsström, 2019).

The Visible Thinking Routine in Written Assignments is Not Implemented During Learning Yet

Upon examining the written responses and conducted interviews, along with an analysis of various factors contributing to the invisibility of students' cognitive processes, another noteworthy factor surfaces. The inclination of students to prioritize the correctness of results over documenting solution steps suggests a lack of integration of visible thinking routines into their learning routines. This finding resonates with research by Al-Abdullatif and Alsaeed (2019), which underscores that visible thinking routines in teaching have demonstrated efficacy in enhancing student achievement and learning progress. Moreover, the adoption of visible thinking routines holds the potential to induce transformative changes in classroom practices when compared to conventional teaching methodologies (Wright et al., 2022). Incorporating visible thinking routines empowers teachers to observe students more openly, formulate inquiries based on observations, generate opinions on meanings, analyze problems, confront biases, and ideate for future instructional practices (Kline, 2008).

Several routines contribute to rendering students' thinking visible (Ritchhart & Perkins, 2008; Salmon, 2016). Notable among them are 1) See/Listen, Think, Wonder (STW), 2) "What makes you say that?" (Salmon, 2016), 3) Connect-Extend-Challenge, and 4) Compass Points. All these routines share a common objective of training students to articulate their thoughts effectively, both orally and in writing. Salmon (2016) observes a gap in some schools, where the emphasis on teaching students critical thinking is insufficient, with more focus on content and outcomes. Consequently, students may possess a superficial understanding of numerous topics, while their comprehension of certain subjects is more profound. Success for students necessitates not only foundational knowledge and skills but also the capacity to engage in critical thinking about their knowledge base.

CONCLUSION

The predominant factor contributing to the obscurity of thinking processes in students with high mathematical resilience, particularly when engaging with numeracy-type problems, is the entrenched habit of prioritizing the correctness of results. During the problem-solving phase, these students frequently omit the documentation of solution steps on the answer sheet, favoring the final outcome over the intricacies of the solution process. This habitual inclination can hinder the cultivation of students' thinking processes, as it tends to sideline the deeper understanding and analysis of mathematical problems. Additionally, students' prior exposure to diverse problem types in the classroom setting enhances their proficiency in problem-solving. This competence is further refined through the application of previous knowledge and an understanding of foundational mathematical concepts. Here, conceptual understanding pertains to the assimilation of previous mathematical concepts that can be leveraged in solving mathematical problems. The teacher's role plays a significant part in shaping students' thinking processes. Teachers who are less inclined to provide feedback on students' tasks, particularly concerning the solution process, significantly impact the development of students' thinking abilities. The incorporation of visible thinking routines into written tasks during the learning process holds substantial promise in training students to effectively articulate their thinking processes, provided these routines are implemented with efficacy.

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AUTHOR CONTRIBUTIONS

Each author participated in the conception and design of the study, data collection, analysis, interpretation, writing, and revising of the manuscript. Additionally, all authors have given their approval for the final version of the manuscript.

CONFLICT OF INTEREST STATEMENT

The authors assert that they do not have any conflicts of interest related to the research, writing, and publication of this paper.

REFERENCES

Al-Abdullatif, A. M., & Alsaeed, M. S. (2019). Evaluating visible learning: Mathematics teachers' practices in technology-enhanced classrooms. *Cogent education*, *6*(1), 1686798. https://doi.org/10.1080/2331186X.2019.1686798

Albay, E. M., & Eisma, D. V. (2021). Performance task assessment supported by the design thinking process: Results from a true experimental research. *Social Sciences & Humanities Open*, *3*(1), 100116. https://doi.org/10.1016/j.ssaho.2021.100116

Alexander, P. A., Zhao, H., & Sun, Y. (2020). Spontaneous focusing on what and why? What children's imprecise responses reveal about their mathematical thinking and relational reasoning. *Mathematical Thinking and Learning*, *22*(4), 332–350. https://doi.org/10.1080/10986065.2020.1818471

Amador, J. M., Rogers, M. A. P., Hudson, R., Phillips, A., Carter, I., Galindo, E., & Akerson, V. L. (2022). Novice teachers' pedagogical content knowledge for planning and implementing mathematics and science lessons. *Teaching and Teacher Education*, *115*, 103736. https://doi.org/10.1016/j.tate.2022.103736

As'ari, A. R., Kurniati, D., & Subanji, S. (2019). Teachers expectation of students' thinking processes in written works: A survey of teachers' readiness in making thinking visible. *Journal on Mathematics Education*, *10*(3), 409–424. https://doi.org/10.22342/jme.10.3.7978.409-424

Attami, D., Budiyono, B., & Indriati, D. (2020). The mathematical problem-solving ability of junior high school students based on their mathematical resilience. *Journal of Physics: Conference Series*, *1469*(1), 012152. https://doi.org/10.1088/1742-6596/1469/1/012152

Brooks, C., Burton, R., van der Kleij, F., Ablaza, C., Carroll, A., Hattie, J., & Neill, S. (2021). Teachers activating learners: The effects of a student-centred feedback approach on writing achievement. *Teaching and Teacher Education*, *105*, 103387. https://doi.org/10.1016/j.tate.2021.103387

Burgess, S., Hauberg, D. S., Rangvid, B. S., & Sievertsen, H. H. (2022). The importance of external assessments: High school math and gender gaps in STEM degrees. *Economics of Education Review*, *88*, 102267. https://doi.org/10.1016/j.econedurev.2022.102267

Creswell, J. W. (2009). *Research design: Qualitative, quantitative, and mixed methods approaches*. Thousand Oaks, CA: Sage Publication.

Czocher, J. A. (2016). Introducing modeling transition diagrams as a tool to connect mathematical modeling to mathematical thinking. *Mathematical Thinking and Learning*, *18*(2), 77–106. https://doi.org/10.1080/10986065.2016.1148530

de Vries, J. A., Dimosthenous, A., Schildkamp, K., & Visscher, A. J. (2022). The impact on student achievement of an assessment for learning teacher professional development program. *Studies in Educational Evaluation*, *74*, 101184. https://doi.org/10.1016/j.stueduc.2022.101184

Fatimah, A. E., & Fitriani, F. (2021). Analisis kemampuan berpikir kritis matematis ditinjau dari resiliensi matematis mahasiswa pendidikan teknik informatika dan komputer. *Journal of Didactic Mathematics*, *2*(2), 94–100. https://doi.org/10.34007/jdm.v2i2.871

Ferrini-Mundy, J. (2000). Principles and standards for school mathematics: A guide for mathematicians. *Notices of the American Mathematical Society*, *47*(8), 868–876.

Forsström, S. E. (2019). Role of teachers in students' mathematics learning processes based on robotics integration. *Learning, Culture and Social Interaction*, *21*, 378–389. https://doi.org/10.1016/j.lcsi.2019.04.005

Fraser, S., Beswick, K., & Crowley, S. (2019). Making tacit knowledge visible: Uncovering the knowledge of science and mathematics teachers. *Teaching and Teacher Education*, *86*, 102907. https://doi.org/10.1016/j.tate.2019.102907

Glenn, D. E., Demir-Lira, Ö. E., Gibson, D. J., Congdon, E. L., & Levine, S. C. (2018). Resilience in mathematics after early brain injury: The roles of parental input and early plasticity. *Developmental Cognitive Neuroscience*, *30*, 304–313. https://doi.org/10.1016/j.dcn.2017.07.005

Gulkilik, H., Moyer-Packenham, P. S., Ugurlu, H. H., & Yuruk, N. (2020). Characterizing the growth of one student's mathematical understanding in a multi-representational learning environment. *Journal of Mathematical Behavior*, *58*, 100756. https://doi.org/10.1016/j.jmathb.2020.100756

Hanney, R. (2018). Problem topology: Using cartography to explore problem solving in student-led group projects. *International Journal of Research and Method in Education*, *41*(4), 411–432. https://doi.org/10.1080/1743727X.2017.1421165

Hull, T. H., Balka, D. S., & Miles, Ruth, H. (2011). *Visible thinking in the K-8 mathematics classroom*. Newbury Park, CA: Corwin Press Inc.

Hwang, G. J., Wang, S. Y., & Lai, C. L. (2021). Effects of a social regulation-based online learning framework on students' learning achievements and behaviors in mathematics. *Computers and Education*, *160*, 104031. https://doi.org/10.1016/j.compedu.2020.104031

Johnston-Wilder, & Lee, C. (2010, September 1-4). *Developing mathematical resilience*. Paper presented at the BERA Annual Conference 2010, University of Warwick, Coventry, England. Retrieved from https://oro.open.ac.uk/24261/2/3C23606C.pdf

Kim, J. Y., & Lim, K. Y. (2019). Promoting learning in online, ill-structured problem solving: The effects of scaffolding type and metacognition level. *Computers and Education*, *138*, 116–129. https://doi.org/10.1016/j.compedu.2019.05.001

Kline, L. S. (2008). Documentation panel: The "making learning visible" project. *Journal of Early Childhood Teacher Education*, *29*(1), 70–80. https://doi.org/10.1080/10901020701878685

Komala, E. (2018). Mathematical resilience mahasiswa pada mata kuliah Struktur Aljabar I menggunakan pendekatan explisit instruction integrasi peer instruction. *Mosharafa: Jurnal Pendidikan Matematika*, *6*(3), 357–364. https://doi.org/10.31980/mosharafa.v6i3.324

Kooken, J., Welsh, M. E., McCoach, D. B., Johnston-Wilder, S., & Lee, C. (2016). Development and validation of the mathematical resilience scale. *Measurement and Evaluation in Counseling and Development*, *49*(3), 217–242. https://doi.org/10.1177/0748175615596782

Kurnia, H. I., Royani, Y., Hendiana, H., & Nurfauziah, P. (2018). Analisis kemampuan komunikasi matematik siswa SMP ditinjau dari resiliensi matematik. *Jurnal Pembelajaran Matematika Inovatif*, *1*(5), 933–940. https://journal.ikipsiliwangi.ac.id/index.php/jpmi/article/view/1597/288

Lee, G., & Simpkins, S. D. (2021). Ability self-concepts and parental support may protect adolescents when they experience low support from their math teachers. *Journal of Adolescence*, *88*, 48–57. https://doi.org/10.1016/j.adolescence.2021.01.008

Legesse, M., Luneta, K., & Ejigu, T. (2020). Analyzing the effects of mathematical discourse-based instruction on eleventh-grade students' procedural and conceptual understanding of probability and statistics. *Studies in Educational Evaluation*, *67*, 100918. https://doi.org/10.1016/j.stueduc.2020.100918

Li, Q., Gu, Q., & He, W. (2019). Resilience of Chinese teachers: Why perceived work conditions and relational trust matter. *Measurement*, *17*(3), 143–159. https://doi.org/10.1080/15366367.2019.1588593

Lim, N. G. (2022). Developing high-level thinking. In N. G. Lim, *Clash of the mind and heart: ~~Parents'~~ Playbook for helping youths succeed* (pp. 125–143). Singapore: World Scientific Publishing. https://doi.org/10.1142/9789811252617_0009

Lutovac, S. (2019). Pre-service mathematics teachers' narrated failure: Stories of resilience. *International Journal of Educational Research*, *98*, 237–244. https://doi.org/10.1016/j.ijer.2019.09.006

Mason, J., Burton, L., & Stacey, K. (2010). *Thinking mathematically* (2nd ed.). London, England: Pearson Education.

Möhring, W., Ribner, A. D., Segerer, R., Libertus, M. E., Kahl, T., Troesch, L. M., & Grob, A. (2021). Developmental trajectories of children's spatial skills: Influencing variables and associations with later mathematical thinking. *Learning and Instruction*, *75*, 101515. https://doi.org/10.1016/j.learninstruc.2021.101515

Musich, S., Wang, S. S., Schaeffer, J. A., Kraemer, S., Wicker, E., & Yeh, C. S. (2022). The association of increasing resilience with positive health outcomes among older adults. *Geriatric Nursing*, *44*, 97–104. https://doi.org/10.1016/j.gerinurse.2022.01.007

Novitasari, M., Sutama, Narimo, S., Fathoni, A., Rahmawati, L., & Widyasari, C. (2020). Habituation of digital literacy and critical thinking in mathematics in elementary school. *International Journal of Scientific and Technology Research*, *9*(3), 3395–3399. https://doi.org/10.31004/basicedu.v6i4.3173

Perry, M., Bates, M. S., Cimpian, J. R., Beilstein, S. O., & Moran, C. (2022). Impacting teachers' reflection on elementary mathematics classroom videos in online asynchronous professional learning contexts. *Teaching and Teacher Education: Leadership and Professional Development*, *1*, 100003. https://doi.org/10.1016/j.tatelp.2022.100003

Pitts, C., Anderson, R., & Haney, M. (2018). Measures of instruction for creative engagement: Making metacognition, modeling and creative thinking visible. *Learning Environments Research*, *21*(1), 43–59. https://doi.org/10.1007/s10984-017-9238-9

Pusat Asesmen dan Pembelajaran. (2020). *Desain pengembangan soal AKM*. Jakarta, Indonesia: Badan Penelitian dan Pengembangan dan Perbukuan Kementerian Pendidikan dan Kebudayaan.

Rahmatiya, R., & Miatun, A. (2020). Analisis kemampuan pemecahan masalah matematis ditinjau dari resiliensi matematis siswa SMP. *Teorema: Teori dan Riset Matematika*, *5*(2), 187–202. https://doi.org/10.25157/teorema.v5i2.3619

Ritchhart, R., & Perkins, D. (2008). Making thinking visible. *Educational Leadership*, *65*(5), 57–61. https://doi.org/10.3115/1599600.1599641

Rosen, Y., & Tager, M. (2014). Making student thinking visible through a concept map in computer-based assessment of critical thinking. *Journal of Educational Computing Research*, *50*(2), 249–270. https://doi.org/10.2190/EC.50.2.f

Røykenes, K. (2016). "My math and me": Nursing students' previous experiences in learning mathematics. *Nurse Education in Practice*, *16*(1), 1–7. https://doi.org/10.1016/j.nepr.2015.05.009

Salmon, A. (2016). *How visible thinking enhances children's learning*. Lincoln, NE: Exchange Press, Inc.

Sheppard, M. E., & Wieman, R. (2020). What do teachers need? Math and special education teacher educators' perceptions of essential teacher knowledge and experience. *Journal of Mathematical Behavior*, *59*, 100798. https://doi.org/10.1016/j.jmathb.2020.100798

Stylianides, A. J., & Stylianides, G. J. (2022). Introducing students and prospective teachers to the notion of proof in mathematics. *Journal of Mathematical Behavior*, *66*, 100957. https://doi.org/10.1016/j.jmathb.2022.100957

Sugiyono, S. (2016). *Metode penelitian kuntitatif, kualitatif dan R&D*. Bandung, Indonesia: Alfabeta.

Supriadi, D., Mardiyana, M. & Subanti, S. (2015). Analisis proses berpikir siswa dalam memecahkan masalah matematika berdasarkan langkah Polya ditinjau dari kecerdasan emosional siswa kelas VIII SMP Al Azhar Syifa Budi tahun pelajaran 2013/2014. *Jurnal Elektronik Pembelajaran Matematika*, *3*(2), 204–214.

van Dooren, W., de Bock, D., Vleugels, K., & Verschaffel, L. (2010). Just answering . . . or thinking? Contrasting pupils' solutions and classifications of missing-value word problems. *Mathematical Thinking and Learning*, *12*(1), 20–35. https://doi.org/10.1080/10986060903465806

van Garderen, D., Lannin, J. K., & Kamuru, J. (2020). Intertwining special education and mathematics education perspectives to design an intervention to improve student understanding of symbolic numerical magnitude. *Journal of Mathematical Behavior*, *59*, 100782. https://doi.org/10.1016/j.jmathb.2020.100782

Wardhani, W. A., Subanji, S., & Dwiyana, D. (2016). Proses berpikir siswa berdasarkan kerangka kerja Mason. *Jurnal Pendidikan: Teori, Penelitian, dan Pengembangan*, *1*(3), 297–313. https://doi.org/10.17977/jp.v1i3.6152

Wright, P., Carvalho, T., & Fejzo, A. (2022). Visible mathematics pedagogy: A model for transforming classroom practice. *Educational Action Research*, *30*(2), 168–191. https://doi.org/10.1080/09650792.2020.1850497

Yeager, D. S., & Dweck, C. S. (2012). Mindsets that promote resilience: When students believe that personal characteristics can be developed. *Educational Psychologist*, *47*(4), 302–314. https://doi.org/10.1080/00461520.2012.722805