# ANALYSIS AND UNRAVELING MISCONCEPTIONS: EXAMINING TENTH-GRADE HIGH SCHOOL STUDENTS' ERRORS IN HOTS QUESTIONS ON EXPONENTIAL EQUATIONS AND EFFECTIVE SCAFFOLDING STRATEGIES

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ARTICLE INFO	ABSTRACT					
Article History:Received20/04/2023Revised15/06/2023Approved22/08/2023Published07/09/2023	The objective of this study is to elucidate the errors made by tenth-grade students at SMA Hati Bilingual Boarding School Probolinggo, Indonesia, when tackling Higher Order Thinking Skills (HOTS) questions on exponential equations, along with the scaffolding process employed to mitigate these errors. Employing qualitative research methods, four participants were chosen based on their performance in completing HOTS questions on exponential equations, followed by in-depth interviews. The identified student errors encompass basic errors, appropriate errors, missing information, and partial insights. The analysis aligns these errors predominantly with Brodie's error taxonomy. Furthermore, the study highlights the efficacy of instructional scaffolding, utilizing the Roehler and Cantlon model. Future research avenues could diversify by exploring students' error patterns in addressing HOTS questions through alternative theoretical frameworks, thereby enriching pedagogical strategies aimed at optimizing educational outcomes.					
<b>Keywords:</b> Error analysis HOTS question Exponential equation Brodie error taxonomy Scaffolding						

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## **INTRODUCTION**

Mathematics assumes a pivotal role in educational advancement, fostering the refinement of human cognitive capacities. Its significance lies in its capacity to cultivate logical and critical thinking skills (Yıldırım & Uzun, 2021). Despite its paramount importance, mathematics often garners disfavor among students due to its perceived complexity. Consequently, many encounter challenges in comprehending mathematical concepts during their educational journey. Hasibuan (2018) contends that such difficulties stem from a lack of clarity regarding the objectives and content of the curriculum. This ambiguity manifests in the form of errors when students engage with mathematical tasks and inquiries (Latifah & Afriansyah, 2021). Elucidating this, Wijaya and Masriyah (2013) define errors as deviations from established correctness or consensus. In the context of problem-solving, students commonly err in grasping the essence of mathematical queries, understanding formulas (Cahyani & Sutriyono, 2018), executing solution methodologies, and formulating conclusions or rationales (Ainin, 2020). Building upon this foundation, research endeavors by Siregar (2019), Nur'aini and Munandar (2021), and Marasabessy et al. (2021) corroborate the prevalence of conceptual misunderstandings among students when confronted with mathematical challenges.

Research by Subanji and Nusantara (2013) and Usodo et al. (2020) posit that the errors students commit during mathematical problem-solving warrant immediate attention to prevent potential encumbrances to their comprehension of forthcoming concepts. To mitigate these errors effectively, conducting error analysis becomes imperative, with its primary objective being the identification of error types and the formulation of corresponding remedial strategies (Aliah & Bernard, 2020). This perspective aligns with the assertions of Brown and Skow (2016) and Rahmania and Rahmawati (2016), who delineate error analysis as a diagnostic assessment tool for teachers to discern the nature and causes of students' errors, thus facilitating targeted interventions. The error analysis



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procedure entails administering test questions aligned with the curriculum content. Subsequent examination of students' mistakes serves as a diagnostic tool, offering insights into both the depth of understanding and the precise loci of misconceptions. Consequently, teachers can devise tailored approaches to rectify these errors and fortify students' conceptual mastery.

The findings of a preliminary investigation by Putri et al. (2023) elucidate that High Order Thinking Skills (HOTS) questions serve as tools for assessing advanced cognitive abilities. Consequently, further exploration is warranted to scrutinize students' errors when tackling HOTS questions pertaining to exponential equations. This research endeavors to conduct error analysis guided by Brodie's theoretical framework. Brodie (2009) categorizes errors into four types. First, basic errors, stemming from misconceptions or lack of understanding of fundamental concepts. Second, appropriate errors, arising when students grasp basic concepts but falter in application. Third, missing information errors, occurring when students comprehend the problem and relevant concepts but fail to synthesize a complete solution. Fourth, partial insight errors, attributable to lapses in attentiveness during solution processing, leading to inadvertent mistakes or calculation errors.

To address student errors encountered when tackling HOTS questions related to exponential equations, scaffolding emerges as a viable solution. Scaffolding, within the realm of learning activities, serves as a conduit, bridging students' existing knowledge with new concepts. Fatahilah et al. (2017) delineates scaffolding as the pedagogical practice wherein teachers offer guidance to students, gradually withdrawing support as students gain autonomy in problem-solving. In this study, scaffolding strategy draws from the framework proposed by Roehler and Cantlon (1997), encompassing five key components. (1) Offering explanations. Teachers furnish explicit elucidations regarding the content being studied, elucidating its significance, applicability, and implementation. (2) Inviting student participation. Teachers actively engage students, providing avenues for them to participate actively throughout the learning process. (3) Verifying and clarifying student understandings. Teachers assess students' comprehension levels through verification and clarification exercises, ensuring clarity of concepts. (4) Modeling desired behaviors. Teachers exemplify the desired cognitive and behavioral processes, demonstrating how individuals should think, feel, and act in various contexts. (5) Inviting students to contribute clues. Teachers encourage students to actively seek and provide cues or hints pertinent to task completion, fostering independent problem-solving skills. By employing these scaffolding strategies, educators can scaffold students' learning experiences, fostering gradual independence and proficiency in tackling HOTS questions on exponential equations.

## METHOD

The primary objective of this study is to scrutinize the errors made by students while tackling Higher Order Thinking Skills (HOTS) questions related to exponential equations and to offer appropriate scaffolding interventions. Employing a qualitative methodology aligns with Creswell (2009), emphasizing the scrutiny of data for descriptive insights and employing textual analysis to discern the broader implications of the findings. The research apparatus encompasses a test sheet featuring two HOTS questions on exponential equations, an interview guide, and a scaffolding implementation manual. Four students, meticulously chosen from an initial cohort of 18, serve as subjects, each representing distinct error categories: basic errors, appropriate errors, missing information, and partial insight. Data acquisition employs a combination of testing and interview methodologies. The examination gauges the nature of errors exhibited by students when confronted with HOTS questions on exponential equations. Subsequently, through interviews, the researcher corroborates the accuracy of student responses and elucidates misconceptions. Employing the Roehler and Cantlon (1997) strategy, the researcher then implements scaffolding techniques aimed at mitigating student errors. The resultant data encompasses detailed accounts of student responses to test queries, alongside transcripts of interviews and the scaffolding process.

### RESULTS

The analysis of responses elicited from 18 tenth-grade students enrolled at SMA Hati Bilingual Boarding School, located in Probolinggo Regency, Indonesia, pertaining to their engagement with test items revealed a consistent pattern of errors across all participants when tackling Higher Order Thinking Skills (HOTS) questions within the topic of exponential equations. Despite exhibiting a variety of error types, each student demonstrated lapses in comprehension or application of key concepts. Detailed error profiles for the cohort are delineated in Table 1 for comprehensive elucidation and further investigation.

Based on the findings delineated in Table 1, the analysis reveals that among the 18 students examined, 8 exhibited errors categorized as basic error, 7 demonstrated errors classified as appropriate error, 8 showcased errors characterized by missing information, and 3 showcased errors falling under the partial insight category. Subsequently, the researcher select 4 students, each exemplifying a distinct error type, to serve as focal subjects for further investigation. These selected students, denoted by the codes Subject 1 (MJ), Subject 2 (MF), Subject 3 (MA), and Subject 4 (DM), underwent detailed examination regarding their respective error profiles and received targeted instructional scaffolding tailored to address their specific needs.

Catagory	Student															Total			
Category	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	Total
Basic Error																			8
Appropriate Error																			7
Missing Information					$\checkmark$														8
Partial Insight																			3

Table 1. Results of identifying student errors.

## Error Analysis and Scaffolding Process Description Subject 1 (MJ)

Following the analysis of responses and interview findings, Subject 1 (MJ) errors in questions 1 and 2 predominantly stem from a basic error category, attributed to a misunderstanding of the problem statement. MJ's problem-solving approach involves attempting to substitute values that satisfy the equation, as outlined in Figure 1. Herein, we outline the scaffolding process tailored to address MJ's error in question number 1.

- R : Let's revisit question number 1. Take another look at it.
- MJ : Sure thing, ma'am. (MJ reads question number 1 aloud)
- R : How many similarities do you see between the questions?
- MJ : There are two similarities, ma'am.
- R : What form do these two equations take in the explained question?
- MJ : They form a system of equations, ma'am.
- R : Alright, let's tackle solving this system. We'll start by using  $2^x$  and  $3^y$  as examples. Can you create an example?
- MJ : Let's say  $2^x$  equals *a* and  $3^y$  equals *b*.
- R : Now, write new equations based on these examples.
- MJ : What about the second equation, ma'am? It involves  $3^{(j+1)}$ . (MJ proceeds to formulate a new equation for the first equation).
- R : Remember the exponent rule  $a^{(m+n)} = a^m \cdot a^n$ . How does this apply to  $3^{(r+1)}$ ?
- MJ : Um, that means  $3^{(y+1)} = 3^y \cdot 3^1$ . Is that correct, ma'am?
- R : Exactly. So, what is the form of the new equation for the second equation?
- MJ : So,  $a b \cdot 3^1 = -11$ , right ma'am?
- R : Yes, that's correct. Can you simplify it further? What is the value of  $3^1$ ?
- MJ : 3, ma'am. So, is this the new equation, ma'am? (MJ writes down the new equation accurately)
- R : Do you remember how to solve the system of equations?
- MJ : I believe I do, ma'am. I'll give it a shot. (MJ proceeds to solve the system of equations using the elimination and substitution method)
- MJ : I've found a = 16 and b = 9, ma'am.
- R : Excellent. Now, let's revisit the example you previously created. Why is that?
- MJ : Like this, ma'am. (MJ presents the results in writing)
- R : This form represents an exponential equation. Can you determine the values of x and y from the equation?
- MJ : Not quite yet, ma'am. I'm not sure how to proceed.
- R : (The researcher explains how to solve exponential equations). Can you solve the exponential equation now?
- MJ : I'll give it a shot, ma'am. (MJ proceeds to solve the exponential equation)
- R : Well done. You've determined the values of x and y. Is that the final answer?
- MJ : (MJ re-reads question number 1). You're actually asked to find the solution set, ma'am.
- R : Please write down the solution set.
- MJ : My apologies, ma'am, I forgot.
- R : (The researcher provides an example of writing a solution set). What is the solution set for problem number 1?
- MJ : So, it looks like this, ma'am. (MJ writes down the solution set)
- R : That's correct. Great work.

The scaffolding process facilitates MJ's comprehension of the question's nuances, thereby enhancing MJ's capacity to furnish a more accurate response to question number 1. Additionally, this process imbues MJ with a deeper understanding of the methodology involved in solving exponential equations, thereby equipping MJ with the requisite skills to navigate the complexities inherent in addressing question number 1. Moreover, the scaffolding process extends to question number 2, leveraging the foundational comprehension acquired during the scaffolding intervention for question number 1. Consequently, MJ is better poised to grasp the intricacies of question number 2 and rectify any errors therein, drawing upon the insights gleaned from the preceding scaffolding endeavor.



Figure 1. Description of MJ's answer.

## Error Analysis and Scaffolding Process Description Subject 2 (MF)

Based on the results of the error analysis, MF exhibited appropriate error types in both questions 1 and 2, indicative of a lack of understanding in solving exponential equations, resulting in erroneous responses. Further details regarding MF's answers and errors are delineated in Figure 2. Herein, we outline the scaffolding process tailored to address MF's error in question number 1.

R	:	Take another look at the explanation you provided for your answer. Can you walk me through how you arrived at it?
MF	:	Of course, ma'am. (MF proceeds to explain their answer)
R	:	Are the calculations you performed during the elimination process accurate?
MF	:	(MF reviews their work) Yes, ma'am, they are.
R	:	Is $-a - (-3) = -4a$ correct?
MF	:	Oh, no, ma'am. My mistake.
R	:	Take another shot at refining your answer.
MF	:	Yes, ma'am. Let me correct that. So, $a = 9$ and $b = 16$ .
R	:	Good. Now, let's revisit the example you generated. Can you write it down?
MF	:	(MF writes out the corrected answer) There we go, ma'am.
R	:	Excellent. Now, let's tackle solving the exponential equation you've written. Earlier, you said $2^x = 8$ implied $x = 8$ , but
		that turned out to be incorrect, right?
MF	:	Yes, ma'am.
R	:	What is the correct answer?
MF	:	Um, it's 3, ma'am.
R	:	How did you arrive at 3?
MF	:	Because $2^3 = 8$ , ma'am.
R	:	Right. (The researcher provides guidance on solving exponential equations and gives examples). Now, given the corrected values of a
		and b, can you find x and y?
MF	:	Like this, ma'am? (MF presents the solution)
R	:	Not quite. Can you solve exponential equations? Is this the final result?
MF	:	Yes, ma'am, I can. The question asks for the solution set, ma'am.
R	:	Do you remember how to write the solution set?
MF	:	Yes, it's $HP = (4, 2)$ . Right, ma'am?
R	:	Almost there, just add curly braces around it.
MF	:	Ah, got it, like this, right? (MF shows the corrected answer)
R	:	Well done.

Through the scaffolding process, MF can acquire a deeper understanding of solving exponential equations, thereby facilitating improvement in his response to question number 1. Subsequently, the scaffolding process extends to question number 2, enabling MF to address and rectify any errors encountered. Leveraging the insights gained from the scaffolding intervention for question number 1, MF is equipped with the necessary knowledge and strategies to correct his mistakes in question number 2 as well.



Figure 2. Description of MF's answer.

## Error Analysis and Scaffolding Process Description Subject 3 (MA)

Subject 3 (MA) errors in question 1 entail a missing information type, manifesting as an incomplete response, while in question 2, the error pertains to the failure to utilize all available information, leading to an incorrect solution. Comprehensive depiction of MA's answers and errors is presented in Figure 3. Herein, we outline the scaffolding process tailored to address MA's error in question number 1.

- R : In your answer, you haven't included the solution set, right?
- MA : Is it not this, ma'am? (MA points to the x and y values obtained)
- R : No, that's not correct. Have you forgotten how to write a solution set?
- MA : If it's not that, is it like this, ma'am? HP = (x = 4, y = 2).
- R : You don't need to include the variables. It should look like this. (The researcher provides an example)
- MA :  $HP = \{(4, 2)\}$ . Is this correct, ma'am?
- R : Yes, that's right.

Through the scaffolding process, MA can enhance his answer to question number 1 with minimal assistance in writing down the solution set. Subsequently, the scaffolding process for question number 2 involves prompting MA to carefully consider the given information, specifically  $x_1 > x_2$ , enabling MA to promptly correct his response.

## Error Analysis and Scaffolding Process Description Subject 4 (DM)

Subject 4 (DM) error profile in question 1 also reflects a missing information type, evident in an incomplete response. In question 2, a partial insight type error is observed, attributed to a lack of carefulness during the problem-solving process. A thorough portrayal of DM's answers and errors is presented in Figure 4. The scaffolding process for question number 1 was similar to that used for MA.



Figure 3. Description of MA's answer.



Figure 4. Description of DM's answer.

The scaffolding process for DM to resolve the errors in question number 2.

- R : Try to explain the answer you wrote.
- DM : Yes, ma'am. (DM explains the answer)
- R : In the section where you wrote  $2^4 = 8$ , you mentioned that you weren't careful enough. What should it be?
- DM : Yes, it should be  $2^3 = 8$ .
- R : Well, so what's the correct answer?
- DM : (DM writes down the correct answer and shows it). Here it is, ma'am.
- R : Great job.

Through the scaffolding process, DM can identify where the mistakes were made, enabling him to promptly correct his answers.

## DISCUSSION

The study's findings revealed that participants made several errors when solving Higher Order Thinking Skills (HOTS) questions involving exponential equations. These errors were categorized according to Brodie (2009) error typology, which includes basic errors, appropriate errors, missing information, and partial insight. Subsequently, the researcher implemented scaffolding techniques to address and remediate these errors. The subsequent sections provide a detailed analysis of each error type along with the corresponding scaffolding strategies employed.

#### **Basic Error**

Subject 1 (MJ) encountered basic errors, as identified through an analysis of student responses and follow-up interviews. These errors arose because MJ lacked comprehension of both the questions and the underlying concepts necessary for solving them. This aligns with Brodie (2009) explanation that basic errors stem from a fundamental misunderstanding of the concepts required to address the problems and previously learned material.

To address these issues, researchers predominantly employed scaffolding strategies that included providing explicit explanations and posing concept-related questions. Additionally, the researchers encouraged MJ to actively engage in the learning process by identifying keywords in the questions and guiding him to discover clues independently. This participatory approach aimed to bolster MJ's conceptual understanding and enable him to correct his mistakes with the provided guidance.

Providing scaffolding for MJ required more time than for other research subjects due to his initial lack of understanding of both the questions and the related concepts. This is consistent with Andriani et al. (2017) and Masfufah and Afriansyah (2021), who asserted that solving mathematical problems necessitates a solid grasp of concepts as a form of their application. Following the scaffolding intervention, MJ demonstrated improved comprehension of the questions and the associated concepts. He also developed a better understanding of exponential equations and their solutions. This progress was evident both from MJ's acknowledgments during the scaffolding sessions and his ability to correctly solve question 1. Furthermore, MJ's newfound understanding facilitated his approach to solving question 2, indicating that the scaffolding process for question 1 effectively reinforced his conceptual grasp and problem-solving skills.

## Appropriate Error

Subject 2 (MF) encountered appropriate errors, as identified through an analysis of student responses and interviews. MF's errors stemmed from a partial understanding of exponential equations; while MF recognized the form of an exponential equation, he did not grasp the method to solve it. This aligns with Brodie (2009) definition of appropriate errors, which occur when students comprehend basic concepts but make mistakes in more advanced ones. Additionally, Dewi et al. (2021) noted that incomplete understanding of concepts can hinder students' ability to draw accurate conclusions, leading to errors in problem-solving.

The scaffolding strategy to address MF's errors involved verifying and clarifying his understanding by reviewing his responses to questions 1 and 2. For instance, MF initially wrote -a - (-3) = -4a, a calculation mistake he recognized upon clarification. Given his basic conceptual understanding, MF was able to correct this error promptly. However, MF also incorrectly solved the exponential equation  $2^x = 8$  by stating x = 8, indicating a lack of procedural knowledge. To rectify this, researchers provided explicit explanations and modeled the correct method for solving exponential equations. Following the scaffolding intervention, MF demonstrated improved understanding of exponential equations, as evidenced by his ability to solve the equation in question 2 using the insights gained from question 1. This process not only enhanced MF's procedural skills but also deepened his overall conceptual grasp.

## **Missing Information**

The research subjects who experienced missing information errors were Subject 3 (MA) for questions 1 and 2, and Subject 4 (DM) for question 1. These errors were identified through an analysis of student answer descriptions and interviews. Both subjects understood the meaning of the questions and the concepts required to solve them, but their answers were incomplete. MA's error on question 2 resulted from not reading the question carefully, while DM could not complete the solution for question 1 because he forgot how to write the solution set. This aligns with Brodie (2009) explanation that missing information errors occur when students understand the problem or basic concept but cannot process it further, resulting in incomplete answers.

In question 1, both MA and DM misunderstood the solution set, hindering their ability to continue and leading to incorrect answers. This is consistent with Radiusman (2020) and Santosa et al. (2022), who stated that a proper understanding of mathematical concepts is crucial for problem-solving. For question 2, MA's error persisted despite working to the end because he neglected the statement

that  $x_1 > x_2$  due to not reading the problem carefully. Similarly, Wahyuni and Nurhadi (2019) found that students often make missing information errors because they cannot process the answer further. To address these errors, researchers used modeling of desired behaviors by providing examples of how to correctly write solution sets to MA and DM. Additionally, an inviting student participation strategy was employed at the beginning of the scaffolding process, asking the subjects to explain their written answers to identify their mistakes and correct them at the end of the process. After the scaffolding intervention, both subjects remembered how to write the solution set correctly and understood the importance of carefully reading the problem.

## **Partial Insight**

Subject 4 (DM) encountered a partial insight error in question 2, as identified through analysis of answer descriptions and interviews. DM's mistake stemmed from a lack of carefulness in the calculation process, resulting in an incorrect answer. This aligns with Brodie (2009) explanation that partial insight errors occur due to inaccuracies in problem-solving processes, often leading to calculation errors or errors stemming from carelessness.

DM's error in question 2 is evident from the answer description, where  $a = 8 \leftrightarrow 2^x = 2^4$ , with the initial example  $a = 2^x$ . This mistake arose due to DM's lack of carefulness and failure to review the answer after completing the work. According to Krisma and 'Adna (2023), such inaccuracies often occur because students neglect to double-check their answers, a sentiment supported by Hermaini and Nurdin (2020), who found that over 80% of students make mistakes due to a lack of review.

To address DM's errors, the researcher employed a scaffolding strategy involving verifying and clarifying DM's understanding by reviewing answer descriptions. Through clarification, DM could identify errors in the written answers. Additionally, an inviting student participation strategy was implemented, prompting DM to explain their answers initially and correct them afterward. The scaffolding process for DM was brief, as DM already grasped the meaning of the questions and related concepts. Upon realizing their mistakes, DM promptly corrected the answer, highlighting the importance of post-work review.

## CONCLUSION

The findings of this study, coupled with the ensuing discussion, yield two primary conclusions. Firstly, students' errors in tackling Higher Order Thinking Skills (HOTS) questions pertaining to exponential equations predominantly align with Brodie's error taxonomy, encompassing: (1) fundamental errors, characterized by a lack of comprehension in solving HOTS queries involving exponential equations entwined with systems of linear equations in two variables and quadratic equations; (2) contextual errors, where students grasp the format of exponential equations, such as  $2^{x} = 8$ , yet falter in their completion; (3) omissions, whereby students fail to articulate the solution set, rendering their responses incomplete; and (4) lapses in attentiveness, leading to overlooking critical information within the questions, such as  $x_1 > x_2$ . Secondly, the instructional scaffolding, following Roehler and Cantlon model, intervenes adeptly. For fundamental errors, strategies include elucidation, eliciting student insights, and fostering active engagement, resulting in enhanced conceptual grasp and refined responses. Addressing contextual errors involves validating and elucidating student comprehension, alongside behavioral modeling, facilitating improved problem-solving proficiency post-scaffolding. Mitigating omissions entails modeling correct approaches and encouraging student involvement, culminating in the adept inclusion of solution sets. Finally, rectifying partial insights necessitates validating and clarifying student understandings, fostering active participation, enabling students to pinpoint and rectify their misconceptions, thereby bolstering their answers. In this investigation, a prevalent occurrence of basic errors among students was observed, attributable to their unfamiliarity with the HOTS question format. Additionally, it's noteworthy that this study solely concentrates on error analysis utilizing Brodie's theoretical framework. Future research endeavors could expand this scope by exploring students' errors in addressing HOTS questions through alternative theoretical lenses. Such endeavors may shed light on diverse facets of students' comprehension and problem-solving approaches, thereby enriching our understanding of pedagogical strategies tailored to enhance educational outcomes.

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## AUTHOR CONTRIBUTIONS

Each author contributed to the conception and design of the study, as well as to data collection, analysis, interpretation, writing, and revising of the manuscript. Furthermore, all authors have approved the final version of the manuscript.

## CONFLICT OF INTEREST STATEMENT

The authors declare that they have no conflicts of interest related to the research, writing, or publication of this paper.

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