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The Misconceptions of Students on Equations based on Their Arithmetic Knowledge

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ABSTRACT

Many students have an operational view of the equals sign. This conception is seen as a misconceptions. This study aims to investigate the extent to which students have misconceptions about equality, where the equal sign is the focus. This research is a descriptive qualitative research with a case study approach. A total of 35 students in the city of Bandung became participants in this study. The results of the study reported that there were several misconceptions such as closing, using all numbers in the equation, string operations, and “pindah ruas”.

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The relational understanding of the equal sign is one of the important concepts that students must master to build their initial algebraic thinking foundation. The equal sign in the school curriculum is usually not introduced as a relational sign indicating an equal relationship, but as an operational sign (McNeil & Alibali, 2005). Equality describes the 'equal to' relationship between mathematical objects which can be numbers, measurements, shapes, numeric statements, or functions (Vale, 2013). When students are able to understand and use equivalence, they can pay attention to these relationships to derive mental strategies for computational operations and to solve problems known as relational thinking (Carpenter, et al., 2003).

The equals sign is not always interpreted as an "equal" sign by students. According to Jones, et al., (2012) elementary school and even college students still tend to use the equals sign as a "sign to do something in the computing process" rather than as an equation sign. Algebra requires a more solid understanding of the meaning and nature of operations than arithmetic. Many students have difficulty learning about algebra because they are unsure about the generally accepted properties of arithmetic. Therefore, one of the early algebraic emphases was building an intuitive awareness of the general properties of number operations. The goal is not to introduce students to formal expressions but to enable students to experience more algebraic thinking. So that the first transition that students need to make to move from the arithmetic method is to approach the problem to the algebraic method by solving equations (Napaphun, 2012). The development of algebraic abilities according to Fuchs (2014) can be done by expanding the relational meaning of the equal sign and including non-standard equations.

Learning an equal sign in an algebraic context will provide a completely different learning experience from learning an equal sign in an arithmetic context in elementary school where usually an equal sign is interpreted as a total number, work results, and an answer to a problem. This understanding of the same sign is often referred to as an operational view (Baiduri, 2015). The operational view is claimed by some researchers (such as Hattikudur & Alibali, 2010; Kieran, 1981, 1992; Knuth, Stephens, McNeil, & Alibali, 2005) to cause difficulties in the transition of arithmetic to algebra. This is because the operational view can hinder the development of the relational view of the equals sign (Mattews, et al., 2010). In addition, there is a large body of literature reporting on the difficulties with the equal sign that many students experience when learning algebra (Carpenter, Franke, & Levi, 2003; Cooper et al., 1997; Herscovics, 1989; Hewwit, 2012; Kieran, 1981; Knuth, Alibali, McNeil, Weinberg, & Stephens, 2005; Knuth, et al., 2006, 2008). These findings are cause for concern because almost any manipulation of equations in algebra requires understanding that the equal sign represents a relation (Carpenter, Franke, & Levi, 2003).

Students' operational conception of the equals sign has been a focal point of reform and research efforts in mathematics education by several international researchers over the past ten years (such as Byrd, et al., 2015; Harbour, Karp & Lingo, 2017; Hewitt, 2012; Jones , et al., 2012; Kindrat & Osana, 2018; Kiziltoprak & Kose, 2017; Meyer, 2016; Stephens, et al., 2013; Vale, 2013; Vermeulen & Meyer, 2017). This shows that the understanding of the equal sign has proven to be a strong 'problem' in various contexts, over a long period of time, and can be experienced by students ranging from elementary school students, middle schools and even university students. However, the equal sign has yet to become a focal point of reform and research efforts in mathematics education in Indonesia. Considering the importance of the relational meaning of the equals sign in understanding rules in an algebraic context, as well as the lack of research reports on the relational conception of the equals sign in mathematics education research in Indonesia, the basis for this research was conducted. The results of this research can help teachers to identify student misconceptions and increase their own knowledge to be able to help students correct these misconceptions before they become entrenched and much more difficult to correct.

METHOD

The case study was chosen because it was considered in accordance with the purpose of this study, namely to explore student misconceptions through an in-depth approach to find out the causes of these misconceptions. Students who were involved as participants in this study were selected using a purposive sampling technique, which was chosen because of its suitability in advancing the research objectives. Grade 7 students in semester 1 were chosen because they were at a stage of transition from arithmetic to algebra. There were 35 students who agreed to participate in the research.

Data collection techniques used are written tests, interviews, and documentation. The written test was used to identify students' misconceptions. The written test questions consist of six descriptive questions which are grouped into two types of questions, namely open numbers and true/false sentences as shown in table 1.

Table 1. Written Test Guidelines

Question Type	Question Description
Open Number	Fill in the value that you think is correct in each of the following boxes. 1. $8 - \square = 10$ 2. $3 + \square = 4 + 12$ 3. $(17 + 4) : 3 = \square + 4$
True/false Sentence	Determine whether the following mathematical sentences are true or false without performing any calculations. Give your reasons. 1. $12 + 2 = 14 : 2 = 7$ 2. $45 + 26 = 47 + 28$ 3. $8 + (3 \times 8) = (5 \times 8) - 8$

Several students who had written test answers that could provide relevant information were selected as resource persons. Each student who was selected as a resource person was given different questions according to their responses. Document analysis was carried out by reading the interview transcripts and analyzing the mathematics textbooks used in the teaching and learning process in class. During the process of collecting, storing, analyzing data, and reporting research results, the confidentiality of the identity of the source is guaranteed.

RESULTS

Analysis of written test results was obtained from written answers which were the result of student work on the questions that had been given. Next, this data is analyzed for right and wrong answers. Answers with correct scores will be analyzed and grouped into operational, relational, and specific categories. While the answers that are considered wrong will be analyzed through misconceptions or experienced by students by looking at student mistakes in working on the questions and then grouped into operational, relational, and specification categories. The term "misconception" is generally used when students' understanding of a concept is considered inconsistent with the meaning in scientific conceptions (Hansen, 2006). Therefore this study focuses on describing the misconceptions that students experience in completing written tests. Aspects of misconception adapted from Meyer (2016), namely: "closing", "using all numbers in the equation", and "string operations". The following is a description of the analysis of students' written test results on each question.

Question 1

Question number one is a question with the type of question "open number sentence questions" to explore information on how students interpret blank signs such as $\square$ before or after the equal sign (with). Generally, students in this problem only succeeded in solving or evaluating arithmetic similarities with the structure "count operations - equal sign (with) - answer" namely a + b = c, including when there is an empty sign such as $\square$". Questions like this are important because they test whether the respondent understands that the variable represents a specific and constant number value (Matthews, et al., 2010). A total of 31 students of class VII SMP the answer is correct. In addition, there were also some students who made mistakes in answering question number 1. The following is an example of an error made by students in solving question number 1. One of the students who answered 18 in question number 1 used the "pindah ruas" method which he demonstrated as in the answer steps given by the following students.

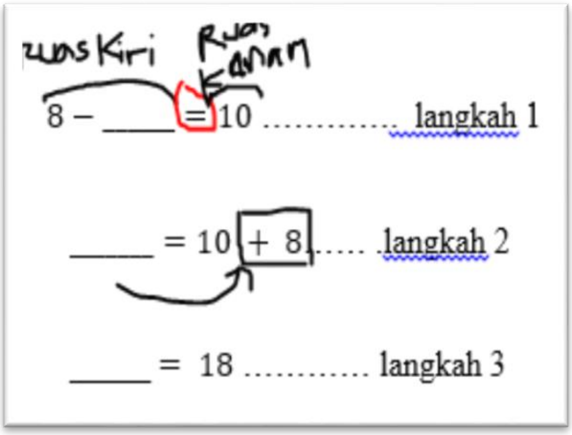

Figure 1. Example of Application of the "Pindah Ruas" Concept in Question 1

In step 2, students “move” the number 8 and the “-” sign from the left side of the equal sign to the right side of the equal sign. If the left side or side of the equal sign is negative (-) then if it shifts to the right side of the equal sign then the value becomes positive (+). This shows that students really understand that the equals sign is a relational symbol so that the two sides can be added, subtracted, divided, or multiplied by the same number which will produce a linear equation that is equivalent to the original linear equation. This shortcut or the fastest or easiest way to solve linear equations with one variable is usually introduced by teachers to students with the term "pindah ruas".

The three categories of student misconceptions in interpreting the equal sign as reported by Meyer (2016), namely closing, using all numbers, and string operations, did not cover all types of misconceptions. The results of this study found student misconceptions using the "pindah ruas" method that did not exist in the category of misconceptions according to Meyer's version, (2016). Therefore, this misconception will be grouped into a new category called the “pindah ruas” category where the category is novelty in this study or the latest findings that have never existed in previous studies.

Question 2

Question number 2 is a question related to the use of the symmetrical property of the equals sign in an arithmetic context as a relational symbol (McNeil, et al., 2017). This is because the equals sign as a mathematical object can be manipulated or represented in various ways (McNeil & Alibali, 2004). The equation in question number 2 is a combination like (a + b = c + d or a + b – c = d + e) which will be difficult to solve correctly without understanding the equal sign as a relational symbol. Figure 2 shows evidence that students have difficulty solving problems so they answer them in an arbitrary way.

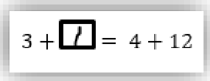

Figure 2. Example of a Misconception of the Closure Category in Question 2

There were 21 students who filled 1 into the box as an answer. The researcher categorizes this error as a closed misconception because students display the approach "the number after the equals sign is the answer for the calculation" and the answer writes 1 as the missing number in this case.

Question 3

Problem number 3 is an equation problem in which there are arithmetic operations on both sides (such as a + b + c = a + ___) which can only be solved if students can understand that doing the same action (limited to the application of known functions, and with certain exceptions, such as division by zero) on each side of the equation preserves the equation as well as the amounts represented by both sides and makes it unnecessary to calculate. Ultimately, students realize that the equals sign represents the relationship between the two sides of the equation and the relationship between the numbers in the two expressions makes calculations unnecessary (Carpenter et al., 2003). However, students tend to interpret expressions in a way that goes against the convention rules of the order of operations (algebraic order) i.e. read them from the left. A total of 27 students filled in the box with 11 as the answer. They expanded the problem by writing 7 in the box and then adding 4 as shown in figure 3.

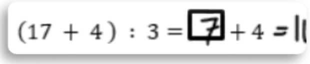

Figure 3. Example of a Misconception of the String Operation Category in Question 3

The error is categorized as a "string operation" misconception where students perceive the equals sign as "results". In the context of question number 4 it is "17+4 produces 21 then 21: 3 produces 7 then 7+4 produces 11". One of the causes of this error is a paradigm shift from the equation of the equals sign in arithmetic which is usually interpreted as "giving" or "result", while in the context of question number 3, the equals sign means "equivalent" so it no longer means "result" or "give". This misconception may not be surprising because many students often hear or see the word "equals" used in everyday language in a way that encourages a sense of causation (Darr, 2003). What's more, in school mathematics, especially when doing arithmetic, the meaning of the equals sign tends to be viewed as an operational sign like 5 + 4 = by reading it as "5 plus 4 makes what?" Even calculators reinforce this meaning by using the equals sign to give computational commands that "make" the answer (Schliemann, et al., 1998).

Question 4

Problem number 4 is a question with the aim of digging up information about mental mathematic abilities or also called mental computation which was introduced by Kindrat & Osana (2018) as an act of performing calculations without using external tools through a series of transformations outside calculating or imagining the steps of a standard algorithm. This ability is important for developing students' relational thinking skills according to Kindrat & Osana because this mental mathematics involves two main processes, namely: (a) choosing a strategy that will make calculations more manageable, and (b) carrying out calculations once a strategy is selected where the type of question or problem. The type of question or question number 4 is "true or false number sentence questions" which can be used to elicit discussion in order to help develop conceptions about the equal sign and students can also be asked to "prove" the statements they make True or false questions according to Vale (2013) are also very useful for overcoming misconceptions and showing the relationship between numbers and operations on both sides. Students give a checklist symbol "(√)" or the letter " (T)" for the statement they think is true and a cross symbol "(×)" or the letter "(F)" for the statement they think is wrong with reasons.

Problem number 4 contains an idea that involves the method that is often used in student work when using the equals sign: namely, as a way of connecting a series of calculations (string operations), for example, 3 + 7 = 10 ÷ 2 = 5 where it is not true in mathematics. Therefore, in question number 4 it is expected that students choose "wrong" as an answer. Tilley (2011) has observed this type of "arbitrary" use of the equals sign as part of a teacher demonstration. According to Sewell (2002) and Risch (2014), many students like the arbitrary use of the equal sign because it does not require dizzying thinking efforts and there is no reconstruction of pre-existing knowledge. The opinion of Sewell (2002) and Risch (2014) seems to be proven in this study.

Question 5

Problem number 5 is a question that is expected to make students successful with equations with large numbers because they can use relationships between expressions, not using computations, and understand the principles of equality (doing the same thing on both sides of the equals sign). Students in this study did not understand what to do because according to them there was no problem to be solved in this number sentence. This is because there are no missing numbers to be found. In addition, they have never encountered a math problem in the form of a "true" or "false" question. There were only 4 students who chose "false" as the answer where the "false" choice was the correct answer. Other students who chose "wrong" as the answer, even though the "wrong" choice was the expected answer in question number 5, they still rely heavily on calculations to make sure the calculation results do not different. Figure 4 below shows the student's strategy of using all the numbers to add up as a basis for drawing conclusions.

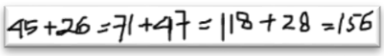

Figure 4. Example of String Operation Misconception in Question 5

Not a few students in this study still interpret the equal sign as an order to perform calculations and obtain answers. So even though they were told to answer without counting, they didn't know what to do so they chose "correct" as the answer because there was no reason to choose "wrong" as an answer. When asked in the interview session, the focus of the students was "getting results" without looking at the relationship between the arithmetic operations and the numbers.

Question 6

To solve problem number 6, students need an understanding of the arithmetic operations of multiplication, division, and equivalence relations. One respondent misunderstood the category of string operations by ignoring parentheses and the order of operations so he answered that the equation was "wrong" because the value on the left is 88 and the right is 32. Figure 5 below shows an example of this misconception.

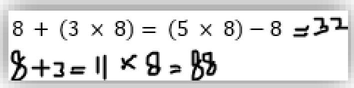

Figure 5. Example of a String Operation Category Misconception in Question 6

Operation sequence errors, especially expressions of the form a ± b × c, are the most common arithmetic errors made by students in high school, college level, and teacher candidates. The parsing obstacle reported by (Gunnarson, Sonnerhed & Hernell, 2016) was the cause of the error. Students tend to read equations like reading sentences that are contrary to conventional rules of order of operations (algebraic order). However, according to Thomas & Tall (2004), it cannot be assumed that every student who has done arithmetic is ready for the notion of more general arithmetic expressions, moreover algebraic notation violates the usual reading order from left to right for example, 2 + 3 × 4 (which is meant to convey the sum of 2 and the result of multiplying 3 by 4) where 3 × 4 must first be calculated which results in 12, then add 12 by 2 to get 14. Often students will read it from left to right as “2 + 3” (which is 5) "multiplied by 4" is 20.

DISCUSSION

Mathematical equality is one of the most important concepts for developing algebraic thinking in school students because it describes a special relationship between mathematical objects in the form of numbers, measurements, shapes, statements of numbers, or functions (McNeil, et al., 2011). Some researchers (such as Alibali, et al, 2007; Knuth et al., 2006; McNeil, et al, 2017) state that a formal understanding of mathematical equality involves students' understanding of the equal sign as a relational symbol. The equals sign can be interpreted as a process as well as a manipulated mathematical object where numbers and expressions can be represented in various ways. Students must have this formal understanding so that arithmetic abilities increase while building the basic foundation for learning algebra such as $3x + 7 = 25$ in high school. Furthermore, McNeil et al., (2017) stated that students' difficulties in mathematical equivalence and their narrow experience with arithmetic in elementary schools strengthened representations, strategies, and concepts that are not easily moved beyond traditional arithmetic such as operations on the left side followed by an equals sign. and a blank answer and a signal to count all numbers as a “total”. Therefore, the formal understanding and correct approach in constructing the concept of mathematical equality deserve more attention from educators.

The equals sign in the school curriculum is usually introduced as an operational sign and not a relational sign of equality. So many research results report that elementary school students have difficulty solving equations, especially equations with operations on both sides with the same sign (for example, $7 + 4 + 5 = 7 + \_\_$), even children still have limited transfer and poor retention from the correct strategy when in middle school. McNeil & Alibali, (2004) claim that excessive practice with arithmetic operations hinders the subsequent learning of more complex equations because children only develop 'routine expertise' with arithmetic operations, but do not understand the concepts underlying what has been learned. Imprecise or poor understanding of arithmetic, even if it leads to a correct response in an arithmetic context, fails to lead to an appropriate response in algebra. Banerjee (2011) mentions that students' prior experiences in arithmetic hardly provide more space for prioritizing representations or there is discussion about the use and need for certain types of notation. In other words, all symbols are understood operationally. Students need to accept that symbols such as “+”, “-” and “=“, have multiple meanings to either indicate a calculation procedure or support a final answer. For example, $x + 3$ is an instruction to do “add 3 to any number and the final answer states the relationship with 3”.

CONCLUSION

Based on the students' performance and misconceptions found, all students who were respondents in this study were only oriented to the equals sign in an operational view where students were accustomed to seeing operations located to the left of the equals sign and numbers located on the right side of the equals sign showing the calculation results. In investigating whether an equation is true, students tend not to really think about the equals sign as a symbol of equality, on the contrary, students only feel the need to do calculations on the left side of the equals sign to get an answer. Therefore, students tend to read the equations from left to right. In other words, students view the equals sign as a sign to "do something" such as calculations. The operational view of the equals sign is still well entrenched in students' minds when they are in high school (Knuth, et al., 2006). Furthermore, Knuth et al. states that the interpretation related to the equal sign can have an important impact or consequence for students in learning algebra. This finding is similar to the results of studies conducted by several previous studies (such as Baiduri, 2015; Gunnarsson, Sonnerhed, & Hernell, 2016; McNeil, et al., 2017).

Students in this study had more difficulty solving questions that did not involve a computational process than questions that involved calculations. Students also generally more difficult to respond to true or false number sentences than open number sentences. In addition, students are more result oriented so they have the obligation to calculate each form of the given equation. This result is in line with the results of several previous studies that "must count" often encourages an operational view of the equals sign (Borenson, 2013; Kiziltoprak & Kose, 2017; Machaba, 2017). Some misconceptions such as closing, using all numbers in equations, string operations, and "pindah ruas" are the impact of students' limited knowledge about the meaning of the equal sign. These findings are some of the important results of this study. Thus, the equals sign is interpreted as an operational rather than a relational sign. The operational conception of the equals sign is largely claimed by some researchers (see Carpenter et al. 2003; Knuth, et al., 2006; Matthews, 2010; McNeil and Alibali, 2004) as a "side effect" that comes from students' learning experiences with symbols in arithmetic that can hinder student achievement in algebra.

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